**Q&A after Sergei Artemov's talk "Justification, Awareness, and Knowledge" at the Workshop "Tsinghua Meets CUNY" October 19, 2018, Beijing.**

**Kamal Lodaya**:
I am not very clear about this. This is the first time that I hear a talk on justification logic. So I am not very clear with the syntax of justifications, because there is +, then there is !, and there is ?, so please does clarify. I'd like to know more about how the syntax is set up.

**Johan van Benthem (chair):**
So this was a question about how the syntax is set up. Maybe, you will also say more, Mel?

**Melvin Fitting**:
It's very flexible, in justification logics before this, you had +, you had *, and maybe other things. Now, you have *, and maybe other things. You need at least that, but you need not to have more than that.

**Sergei Artemov:** The minimal set of operations is $\{s \cdot t\}$ (application only). This suffices to build justification models for Gettier and Russell's examples, but is not enough for realizing modal logics. If we want an explicit version of K, we need to add $"+"$ (sum). For S4 we also need $"!"$ (proof checker), for S5 – the negative verifier "?".

**Johan van Benthem (chair):**
I'm looking around for other questions, and one question which I have is in formalizing philosophical examples. In some sense the level of detail at which you represent them has something to do with your purpose. So what do we learned about these philosophical examples, like the idea or others, so to speak by giving a justification logical analysis. So the analysis itself looks, you know, reasonable and closed to the examples. But this is more a question about what logic does for philosophy. So, once we have that, want do we learned that philosophers didn't see. It's a bit wired to ask questions without answers, but maybe it's also a good practice.

**Melvin Fitting:**
You mentioned philosophy, so there you are.

**Sergei Artemov:** Thank you, Johan, for this question to which you, of course, know the answer, but which provides me with an opportunity to make a generic disclaimer which I had no time to do during the talk.
  *One has to see the forest behind the trees*. Russell's Prime Minister example is not just a witty old joke about "late Prime Minister in 1912" but rather a paradigmatic epistemic scenario in which a true fact has both correct and incorrect justifications. There is no need to explain that this is a basic example from a broad class of epistemic situations related to handling right and wrong justifications, true and fake

news, unreliable intelligence reports, rational and irrational beliefs, etc. Russell's example sets up a challenge to develop a formal logical machinery which fairly represents such epistemic situations. Nowadays, a need for such logical tool has grown far beyond foundations of epistemology.

I am making the case that the traditional modal-based languages fall short of modeling these situations and we need "to go hyperintensional." The model $\mathscr{R}$ introduced in the talk is a representative example of Justification Awareness Models, JAM, which are designed to handle epistemic situations with unreliable evidence. It is a working prototype of JAM for a variety of evidence-based epistemic scenarios, ontologies, data bases, machine learning bases, etc. For epistemology, JAM provide a formalism which closely follows epistemic intuition and this is a virtue of JAM, not a weakness. Intuition becomes a formal logical tool and this, I wish to believe, constitutes a progress in formal epistemology.

**Johan van Benthem (chair):**
Do I see, anyone else. Ah, yeah, Tomoyuki Yamada.

**Tomoyuki Yamada:**
So I think the notion of justification is also relevant for the notion of assertion. So, not only for knowledge but we also require justification for our assertions. So there can be some close relations between if we have something like logic of assertions and then between that logic and justification logics. So if there is any such a thing, I'd be happy to know about it.

**Sergei Artemov:** Foundational works on justification logic have outlined a broad spectrum of interpretations of justifications beyond obvious proof-like objects. This led to numerous papers that studied such interpretations. A practical suggestion: check the references to

> *Sergei Artemov, The Logic of Justification, The Review of Symbolic Logic, 1(4), pp.477-513, 2008*

and you find a variety of follow up papers.

**Fenrong Liu:**
Maybe I ask one.

**Johan van Benthem (chair):**
Yes.

**Fenrong Liu:**
So the relation between this logic and other evidence-based logics. Technical one. So, how Sergei would see other logics, conceptual difference and also technical difference.

**Sergei Artemov:** If I understand the question correctly, you are asking mainly about

evidence logics introduced by van Benthem and Pacuit in 2011. An excellent question, a clear answer to which is given in *van Benthem, Fernandez-Duque, and Pacuit, 2014. Evidence and plausibility in neighborhood structures. Annals of Pure and Applied Logic, 165(1), pp.106-133*:

*"… we end with Sergei Artemov's work on justification logics. … our evidence models provide a coarser level of analysis … . In an evidence model, the agent has evidence for φ provided there is a proposition … that the agent has identified as evidence which logically implies φ. An important feature of justification logic is that a proof term not only records the fact that certain beliefs are grounded in evidence, but also the proof of why that evidence justifies a particular belief."*

I am respectfully adding my comments. **Informally**, we deal here with two related but distinct notions of evidence. For example, in the justification logic paradigm, evidence for the Fermat Last Theorem FLT in Peano Arithmetic PA is a mathematical proof of FLT in PA. In the evidence logic approach, the set of axioms of PA counts as an evidence for FLP. Both of these notions of evidence make sense, the former carries the proof itself whereas the latter tracks assumptions only. From this perspective, *evidence logic is a kind of "justification logic lite"* with some convenient closure properties at the expense of precision.

**Conceptually**, in evidence logic, evidence of $A$ is represented by propositions that logically entail $A$ and so no new logical entities are introduced. In justification logic, evidence is represented by justification terms which are new logical objects, not reduced to propositions.

**Formally**, evidence logic adopts a strong closure property: if evidence yields both $A$ and $A{\to}B$, then it yields $B$ as well. This however comes at a price: the precision of such models is limited, the evidence analysis is not hyperintensional, does not address the logical omniscience issue, and does not distinguish between evidence of $0{=}0$ and of FLT in arithmetic. Justification logic adopts deductive closure not for individual epistemic sets, but rather for the whole collection of them: if $s$ justifies $A{\to}B$ and $t$ justifies $A$, then yet another justification, $s{\cdot}t$, justifies $B$. As a result, justification logic is hyperintensional, controls logical omniscience, and distinguishes proofs of $0{=}0$ and proofs of FLT in PA.

**Technically**, justification logic is capable to tracking assumptions $A,B,C$ … by assigning justification variables to them $x{:}A,\ y{:}B,\ z{:}C$ … and doing the usual justification tracking of $x,y,z,\dots$ . I expect this observation to lead to establishing a formal connection between justification logic and evidence logic. This is an interesting question to study.

Thank you for your attention.