Logic and Existence

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I’m starting with contemporary logic (i.e., late 19th c).
Frege’s view: (a) existence is a 2nd-level concept ($\lambda F \exists x Fx$), (b) names of nonexistents fail to denote, and (c) sentences containing such names are truth-valueless.
Russell’s view: (a) there are no non-existent objects, (b) names of non-existents are really disguised descriptions, and (c) such descriptions can be implicitly defined by existence and uniqueness claims.
But neither view preserves the data:
   “The ancient Greeks worshipped Zeus.”
   This is true, not false or truth-valueless.
   “Jupiter is not identical to Odin” is true, but Frege says it is truth-valueless. And Russell assigns it multiple readings:
   $\neg \exists x \exists y (\ldots x \ldots \& \ldots y \ldots \& x = y)$ (true, but doesn’t have the form $a \neq b$)
   $\exists x \exists y (\ldots x \ldots \& \ldots y \ldots \& x \neq y)$ (false, but intuitively the sentence doesn’t have a false reading).
Beyond Truth-Values: Logical Consequence

- Additionally, there are valid arguments that aren’t preserved by the Frege/Russell view.
- Example:
  - Ponce de Leon searched for the fountain of youth.
  - Therefore, Ponce de Leon searched for something.
  - The conclusion follows on any reading of the premise.
- Compare:
  - The ancient Greeks worshipped Zeus.
    Zeus is a mythical character.
    Mythical characters don’t exist.
    Therefore the ancient Greeks worshipped something that doesn’t exist.
  with:
  - Modern Oregonians worshipped Baghwan Rajneesh.
    Baghwan Rajneesh was a phony religious teacher.
    Phony religious teachers aren’t worthy of worship.
    Therefore, modern Oregonians worshipped something not worthy of worship.

These have similar forms, but theories that treat ‘Zeus’ and ‘Baghwan Rajneesh’ differently can’t preserve common form.
Names of fictional objects (mythical objects, etc.) have a denotation, but we have to figure out what fictions are.

Partial solution: adopt Meinongian interpretation of the quantifier (∃) and use a separate existence predicate (E!) to assert existence, where what exists is a subdomain of what there is.

Such a distinction preserves the following data:

- There are (∃) fictional characters that still inspire us even though they don’t exist (¬E!x).
- Teams of scientists have searched for the Loch Ness monster, but since it doesn’t exist (¬E!x), no one will ever find it.
- All of the characters portrayed in this film are fictitious and any resemblance between them and actual persons (living or dead) is purely coincidental.

And the data and inferences on the previous page can be preserved.
But Quine (1960) says: there is no logical distinction between ‘there is’ and ‘there exists’. To be is to be the value of a variable and that all there is to existence.

So Quine insists $E!x =_{df} \exists y(y = x)$ and raised questions about these notions in modal contexts (i.e., about possibilia).

Quine’s view makes nonsense of how ‘there is’ and ‘there exists’ are used in natural language (cf. previous slide), but he was out to rehabilitate natural language and force compliance to theory.

And logicians subsequently took one of two options:
- Lewis: possible objects exist but aren’t actual.
- Kripke: use models of QML with varying domains and enfeeble the quantifier so that $\diamond \exists x \varphi$ doesn’t imply $\exists x \diamond \varphi$ or vice versa.

The Actualists (Plantinga, Adams, etc.) argued against Lewis (cf. Russell v. Meinong) and argued for variants of Kripke’s view.

Repair Kripke semantics, which compromised the language and axioms of QML (no constants, restricted quantifier laws, etc.)
Simplest QML Defended

- Note Quine’s quantifier (∃) doesn’t imply physical existence. For he asserts that sets exist (since they are required by our best physical theories). So ‘exists’ for Quine is a logical notion.
- Linsky & Zalta 1994 (cf. Williamson 1998): If so, then reinterpret the simplest QML with fixed domains:
  - Ordinary existing objects are concrete (‘E!’) and might not have been concrete, though they exist (in the logical sense) necessarily.
  - Suppose that there exist objects that aren’t concrete but that might have been concrete.
  - Then the Barcan formula is philosophically acceptable:
    ◻∃xφ → ∃x◇φ. Contra Lewis, these contingently nonconcrete objects exist, are actual, but aren’t concrete.
- So the simplest QML (fixed domains, simple logic) has both a possibilist interpretation (∃ = ‘there is’ and E! = ‘exists’) and an actualist interpretation (∃ = ‘there exists’ and E! = ‘concrete’).
- We don’t need to force natural language to conform to theory.
The Problem of Logical Objects

Where do *logical* objects fit in? Objects such as *truth-values*, *possible worlds*, the *extension* of a property, *natural numbers*, *The Triangle*, *fictions*, etc., are not among the possibilia (however conceived).

The reason why is easiest to see in the case of fictional objects.

Kripke’s 1972 [1980] insight: names like ‘Zeus’, ‘Sherlock Holmes’, etc. don’t denote possible objects, since there are too many of these consistent with what is attributed to Zeus and Holmes in their respective stories; we can’t identify a unique denotation on the basis of those stories.

This applies to logical objects generally (including mathematical objects): these objects have only the properties we stipulate them to have (cf. Dedekind).

How can there be such logical objects? How can we account for the existence of logical objects?
The Idea Underlying Object Theory

- Kripke (1973 [2013]): a ‘double usage of predication’: ‘Hamlet was melancholy’ vs. ‘Hamlet was admired by many critics’.
- Plato (Meinwald 1992): (The Form) The Triangle is triangular in some sense of ‘is’, but not in the sense in which ordinary triangles are triangular.
- Boolos (1987): Frege has ‘two kinds of instantiation relation’ ($F \eta x$ and $Fx$). E.g., Uniquely exemplified properties (like $[\lambda x \ x = 1]$) are in the number 1, but 1 ‘falls under’ the property $[\lambda x \ x = 1]$.
- Mally (1912): ‘the round square’ is determined by roundness and squareness without satisfying those properties.
- In my work: Distinguish $xF$ (‘$x$ encodes $F$’) and $Fx$ (‘$x$ exemplifies $F$’), where the latter generalizes to $F^n x_1 \ldots x_n$. Assert: for any combination of properties, there is a abstract (logical) object that encodes just those properties.
First Principles

- Start with simplest 2nd-order QML (fixed domains) with encoding, under Quine’s interpretation of $\exists$, with $E!$ (‘concrete’).
- $\exists F \Box \forall x_1 \ldots x_n (F^n x_1 \ldots x_n \equiv \varphi)$, no free $F^n$ s or $xF$ subformulas in $\varphi$
- Define *ordinary* objects (‘$O!x$’) as:
  - $O!x =_{df} \Diamond E!x$

  Contingently concrete and contingently non-concrete are $O!$.
- Define *abstract* objects (‘$A!x$’) as:
  - $A!x =_{df} \neg \Diamond E!x$

  E.g., Numbers are not the kind of thing that could be concrete.
- Ordinary objects don’t encode properties: $O!x \rightarrow \Box \neg \exists FxF$
- Encoding is modally rigid: $\Diamond xF \rightarrow \Box xF$
- Comprehension: $\exists x (A!x \land \forall F(xF \equiv \varphi))$, for $\varphi$ no free $x$s
- Identity: $x = y =_{df}$
  
  $(O!x \land O!y \land \Box \forall F(Fx \equiv Fy)) \lor (A!x \land A!y \land \Box \forall F(xF \equiv yF))$

  $ix(A!x \land \forall F(xF \equiv \varphi))$ is canonical.
Examples of Logical Objects

- The Triangle.
  \[ \Phi_T = df \xi(A!x & \forall F(xF \equiv \Box \forall y(Ty \to Fy))) \]

- The extension of \( G \).
  \[ \epsilon G = df \xi(A!x & \forall F(xF \equiv \forall y(Fy \equiv Gy))) \]
  Theorem: \( \epsilon F = \epsilon G \equiv \forall x(Fx \equiv Gx) \) (version of Basic Law V)

- \( x \) encodes \( p = df x[\lambda y p] \)

- The truth-value of \( p \)
  \[ p^\circ = df \xi(A!x & \forall F(xF \equiv \exists q(q \equiv p & F = [\lambda y q]))) \]
  Theorem: \( p^\circ = q^\circ \equiv p \equiv q \)

- \( \text{Situation}(x) = df \forall F(xF \to \exists p(F = [\lambda y p])) \)
  \( x \models p \) (\('p \text{ is true in } x') = df \text{Situation}(x) \& x[\lambda y p] \)

- \( \text{PossibleWorld}(x) = df \diamond \forall p((x \models p) \equiv p) \)
  \( \text{Actual}(x) = df \forall p((x \models p) \to p) \)
  Theorem: \( \exists! x(\text{PossibleWorld}(x) \& \text{Actual}(x)) \)
  Theorem: \( \Box p \equiv \forall w(w \models p) \)
  Theorem: \( \diamond p \equiv \exists w(w \models p) \)
Mathematical Objects Are Logical Objects

- The natural number of Gs.
  \[ \#G = df \, ix(A!x \land \forall F(xF \equiv F \approx_E G)) \]
  Theorem: \[ \#F = \#G \equiv F \approx_E G \] (Hume’s Principle)

- Let T be any mathematical theory, and interpret T as a term denoting a situation, so that \( T \models p \) (‘p is true in T’) is defined.

- Let \( \kappa \) be any well-defined singular term in T. Then we can identify \( \kappa \) as a logical object:
  - \( \kappa_T = ix(A!x \land \forall F(xF \equiv T \models F\kappa_T)) \)
  - \( 0_{PNT} = ix(A!x \land \forall F(xF \equiv PNT \models F0_{PNT})) \)
  - \( 0_{ZF} = ix(A!x \land \forall F(xF \equiv ZF \models F0_{ZF})) \)

- In general, we don’t detect the existence of logical objects by a physical information pathway. Their existence is grounded by comprehension and knowledge by acquaintance collapses to knowledge by description.
Defining Logical Existence Using Identity

Quine's Principle ("To be (exist) is to be the value of a variable") is often cashed out as: \( x \text{ exists} \equiv \exists y (y = x) \).

This is particularly useful for languages in which there are terms that might fail to denote, e.g., languages with definite descriptions (\( \exists x \varphi \)), function terms, or complex predicates that don’t denote (e.g., in object theory, \([\lambda x \exists F(xF & \neg Fx)]\)).

Quine’s Principle is then stated as a metadefinition:

- \( \tau \text{ exists} (‘\downarrow’) =_{df} \exists \beta (\beta = \tau) \)

Intuitively, this gives us conditions under which \( \tau \) denotes.

Then the principles of free logic can be stated in 1 of 2 ways:

<table>
<thead>
<tr>
<th>( \forall \alpha \varphi \rightarrow (\exists \beta (\beta = \tau) \rightarrow \varphi^T_\alpha) )</th>
<th>( \forall \alpha \varphi \rightarrow (\tau \downarrow \rightarrow \varphi^T_\alpha) )</th>
</tr>
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<tr>
<td>( \exists \beta (\beta = \tau) )</td>
<td>( \tau \downarrow )</td>
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<tr>
<td>( \varphi \rightarrow \exists \beta (\beta = \tau) )</td>
<td>( \varphi \rightarrow \tau \downarrow )</td>
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Can One Define Logical Existence Using Predication?

- Why define term existence using identity?
- Why not define term existence (i.e., $\tau \downarrow$) by using predication, as follows? (Start with individual terms $\kappa$)
  - $\kappa \downarrow =_{df} \exists F F\kappa$
  - E.g., $\iota x \psi \downarrow =_{df} \exists F \iota x \psi$ (this works fine)
- But in classical logic, this doesn’t generalize from the first- to second-order case. Let $\Pi$ range over relation terms:
  - $\Pi \downarrow =_{df} \exists x \Pi x$
  - E.g., $[\lambda y \varphi] \downarrow =_{df} \exists x ([\lambda y \varphi] x)$ (this fails)
- Unexemplified properties undermines this generalization; so free logic must either have primitive identity or primitive existence.
- Only way to avoid this: use third-order logic (i.e., invoke properties of properties) to define the existence of properties, and so on, ad infinitum.
Existence Can Be Defined in Terms of Predication

- In object theory, there are two kinds of predication and identity is *not* primitive. $\downarrow$ and $=$ can be defined independently and one can then prove $\tau \downarrow \equiv \exists \beta (\beta = \tau)$ as a theorem!
- Existence (by cases):
  - $\kappa \downarrow = df \exists F F \kappa$ (for individual terms $\kappa$)
  - $\Pi \downarrow = df \exists x x \Pi$ (for 1-place relation terms $\Pi$)

  This generalizes for relations using $n$-ary encoding.
- This works because $\vdash \forall F \exists x xF$. 3rd-order logic isn’t necessary.
- Identity (by cases and simplified by object language variables):
  - $x = y = df$
    - $(O!x \land O!y \land \Box \forall F (Fx \equiv Fy)) \lor (A!x \land A!y \land \Box \forall F (xF \equiv yF))$
  - $F^1 = G^1 = df \Box \forall x (xF^1 \equiv xG^1)$

  Relation identity defined in terms of property identity.
- From these definitions it follows, for any term $\tau$, that $\tau \downarrow \rightarrow \tau = \tau$ and that $\tau = \tau' \rightarrow (\tau \downarrow \land \tau' \downarrow)$, which yields:
  - $\tau \downarrow \equiv \exists \beta (\beta = \tau)$
  - 2nd-order logic of encoding (without identity) is self-contained.
Is Logic Committed to Existence Claims?

But is the existence principle for logical objects a part of logic? One’s view may depend on where one went to graduate school.

The early logicists thought logic was committed to existence claims:

- Frege stipulated the existence of two truth-values.
- Frege asserted Law V as a logical principle (at one point, at least, he thought it consistently implied the existence of extensions).
- Whitehead & Russell asserted (implicit) comprehension principles for propositional functions, axiom of reducibility, axiom of infinity, etc.
- Logicians up through Hempel (1945) thought that set theory was part of logic.

Even Carnap (1950) thought one could accept logical and other abstract objects (e.g., propositions, properties, etc.), as long as you distinguish between \textit{internal} vs \textit{external} existence questions.
Logic Implies Existence

- There are a lot of reasons why many logicians now think logic should not imply existence claims. But I think it does.
- Propositional logic is committed to existence claims, since each formulas has a semantic value. If we use propositional variables and quantifiers, we would have, for example:
  - \( p \to (q \to p) \Rightarrow \)
  - \( \forall p \forall q(p \to (q \to p)) \Rightarrow \)
  - \( \exists p \exists q(p \to (q \to p)) \)
- Predicate logic committed to existence, and not just to the non-empty domain of 1st-order logic. Predication involves properties and objects:
  - \( Pa \to \exists F \exists x(Fx) \)

The semantics of 1st-order logic requires predicates to have a semantic value, but the language isn’t expressive enough.
- Modal logic committed to existence: \( \Diamond \varphi \) semantically implies \( \exists w(w \models \varphi) \). But the language isn’t rich enough to express this.
Object Theory Explicitly Implies Existence Claims

- Abstract logical individuals, governed by comprehension:
  \[ \exists x (A!x \land \forall F(xF \equiv \varphi)) \]
  where \( x \) isn’t free in \( \varphi \) (i.e., \( \varphi \) is completely unrestricted)

- This gives the logician the freedom to objectify *any logical pattern*.

- This principle supplies the element missing from Carnap’s view. Only an unrestricted comprehension principle for abstracta guarantees that an arbitrary ‘framework’ has the right semantics so that the internal existence question becomes obviously true.

- Comprehension is logically true in the sense that it is true in all models that contain the objects required for analyzing the truth of (abstract) thoughts and the validity of inferences involving them.
The Picture Generalizes in Type Theory

Russell had multiple reasons for developing relational type theory and they still apply.

Definition of the Types:

- $i$ (type for individuals)
- $\langle t_1, \ldots, t_n \rangle$ (type for relations, where $t_1, \ldots, t_n$ are any types)

When $n = 0$, the type $\langle \rangle$ (‘$p$’) for propositions comes for free.

Type the language:

- $Fx_1 \ldots x_n$ for any $F$ of type $\langle t_1, \ldots, t_n \rangle$ and $x_1, \ldots, x_n$ having types $t_1, \ldots, t_n$
- $xF$ for any $F$ of type $\langle t \rangle$ and $x$ of type $t$

$E!^t$, at every type $t$. So:

- $O!^t x =_{df} \Diamond E!x$
- $A!^t x =_{df} \neg \Diamond E!x$

At every type $t$:

$$\exists x'(A!(^t)x \& \forall F(^t)(xF \equiv \varphi))$$, where $\varphi$ has no free $x$s

Logical objects all the way up!