# Expressivity and Inference in Hybrid Logic

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**Homework Sheet 2**

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**Exercise 1.** Give a tableau proof of $@_i i$.

**Exercise 2.** Give a tableau proof for the following derived rule.

$$\text{Modus Ponens} \qquad \frac{@_i\varphi \quad @_i(\varphi \to \psi)}{@_i\psi}$$

**Exercise 3.** Give a tableau proof of $(\Diamond p \wedge \Diamond\neg p) \to (\Box(q \to i) \to \Diamond\neg q))$. (You may make use of the derived Modus Ponens rule from Exercise 2.)

**Exercise 4.** We say that a frame $(W, R)$ is *convergent* (or *Church Rosser*) iff

$$\forall x \forall y \forall z(Rxy \wedge Rxz \to \exists w(Ryw \wedge Rzw)).$$

Show that the pure hybrid tense logical formula $(Fi \wedge Fj) \to @_j FPi$ defines the class of convergent frames. That is, show (a) that this formula is valid on all convergent frames, and (b) that if a frame is *not* convergent, you can falsify this formula on it.

**Exercise 5.** (a) Give the standard translation of $\Box(\Box i \to i) \to \Box i$. (b) Explain why this formula does *not* define the same class of frames as $\Box(\Box p \to p) \to \Box p$, the Löb formula.