Expressivity and Inference in Hybrid Logic

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Homework Sheet 3

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Exercise 1. Show that the axiom $(i \wedge p) \to @_i p$ is sound. That is, show that for any frame (W, R), for any model $\mathcal{M} = (W, R, V)$ on that frame, and for any world w in W, we have that $\mathcal{M}, w \vDash (i \wedge p) \to @_i p$.

Exercise 2. The two proof rules that let us build named models are:

$$(\text{NAME})\ \frac{\vdash j \to \theta}{\vdash \theta} \qquad\qquad (\text{PASTE})\ \frac{\vdash (@_i \Diamond j \wedge @_j \varphi) \to \theta}{\vdash @_i \Diamond \varphi \to \theta}$$

In both rules, j is a nominal distinct from i that does not occur in φ or θ.

Show that both rules *preserve frame validity.* That is, show that for any frame (W, R):

(a) If $(W, R) \vDash j \to \theta$, then $(W, R) \vDash \theta$; and

(b) If $(W, R) \vDash (@_i \Diamond j \wedge @_j \varphi) \to \theta$, then $(W, R) \vDash @_i \Diamond \varphi \to \theta$.

Exercise 3. Let $\mathcal{M} = (W, R, V)$ be a named model and φ a pure formula. Suppose that for all pure instances ψ of φ, $\mathcal{M} \vDash \psi$. Then $(W, R) \vDash \varphi$.