

# Expressivity and Inference in Hybrid Logic

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## Tableau Rules

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Note: If you want to prove  $\varphi$  using hybrid tableau, you start by prefixing  $\varphi$  by  $\neg@_i$ , where  $i$  is a new nominal not occurring in  $\varphi$ .

### Labelled deduction rules

$$\begin{array}{l}
 \neg \text{ rules} \quad \frac{@_i\neg\varphi}{\neg@_i\varphi} \qquad \frac{\neg@_i\neg\varphi}{@_i\varphi} \\
 \\
 \wedge \text{ rules} \quad \frac{@_i(\varphi \wedge \psi)}{@_i\varphi \quad @_i\psi} \qquad \frac{\neg@_i(\varphi \wedge \psi)}{\neg@_i\varphi \mid \neg@_i\psi} \\
 \\
 \vee \text{ rules} \quad \frac{@_i(\varphi \vee \psi)}{@_i\varphi \mid @_i\psi} \qquad \frac{\neg@_i(\varphi \vee \psi)}{\neg@_i\varphi \quad \neg@_i\psi} \\
 \\
 \rightarrow \text{ rules} \quad \frac{@_i(\varphi \rightarrow \psi)}{\neg@_i\varphi \mid @_i\psi} \qquad \frac{\neg@_i(\varphi \rightarrow \psi)}{@_i\varphi \quad \neg@_i\psi} \\
 \\
 @ \text{ rules} \quad \frac{@_i@_j\varphi}{@_j\varphi} \qquad \frac{\neg@_i@_j\varphi}{\neg@_j\varphi}
 \end{array}$$

### Modality rules

$$\begin{array}{l}
 \diamond \text{ rules} \quad \frac{@_i\diamond\varphi}{@_i\diamond j \quad @_j\varphi} \qquad \frac{\neg@_i\diamond\varphi \quad @_i\diamond k}{\neg@_k\varphi} \\
 \\
 \square \text{ rules} \quad \frac{@_i\square\varphi \quad @_i\diamond k}{@_k\varphi} \qquad \frac{\neg@_i\square\varphi}{@_i\diamond j \quad \neg@_j\varphi}
 \end{array}$$

### Equality rules

$$\frac{(i \text{ occurs on branch})}{@_i i} \qquad \frac{@_i j}{@_j i} \qquad \frac{@_i j \quad @_i \varphi}{@_i \varphi} \qquad \frac{@_j i \quad @_k i}{@_j k} \qquad \frac{@_i \diamond j \quad @_j k}{@_i \diamond k}$$

### Derived rules

$$\text{Modus Ponens} \quad \frac{@_i\varphi \quad @_i(\varphi \rightarrow \psi)}{@_i\psi}$$