Expressivity and Inference in Hybrid Logic

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Homework Sheet 1

The Second Tsinghua Logic Summer School
June 27 – July 3, 2022, Beijing, China

Exercise 1. Give the standard translation of $\Diamond\Diamond i \to \Diamond i$.

Exercise 2. We say that a frame (W, R) is *convergent* (or *Church Rosser*) iff

$$\forall x \forall y \forall z (Rxy \wedge Rxz \to \exists w (Ryw \wedge Rzw)).$$

Show that modal formula $\Diamond\Box p \to \Box\Diamond p$ defines the class of convergent frames. That is, show (a) that this formula is valid on all convergent frames, and (b) that if a frame is *not* convergent, you can falsify this formula on it.

Exercise 3. We say that a frame (W, R) is *antisymmetric* iff

$$\forall x \forall y ((Rxy \wedge Ryx) \to x = y).$$

Show that the pure hybrid formula $@_i\Box(\Diamond i \to i)$ defines the class antisymmetric frames. That is, show (a) that this formula is valid on all antisymmetric frames, and (b) that if a frame is *not* antisymmetric, you can falsify this formula on it. (c) Can you think of another formula not containing @ that defines this class of frames?

Exercise 4. Let $\mathcal{M} = (W, R, V)$ and $\mathcal{M}' = (W', R', V')$ be models for the basic hybrid language (with just one $\Box$ and $\Diamond$), and let Z be a bisimulation-with-constants between $\mathcal{M}$ and $\mathcal{M}'$. Show that for all basic hybrid formulas φ, and all worlds w in $\mathcal{M}$ and w' in $\mathcal{M}'$ such that w is bisimilar to w' we have that:

$$\mathcal{M}, w \models \varphi \text{ iff } \mathcal{M}', w' \models \varphi.$$