Logic, Data, and Incomplete Information

The Second Tsinghua Logic Summer School

Homework Assignment 2
(due Thursday, June 30, 2022 - before class)

Problem 1: We say that a decision problem Q is *semi-decidable* if there an algorithm that produces a list of all "yes" instances of the problem Q. Show that the following statements are equivalent for a decision problem Q:

1. Q is decidable.
2. Both Q and the *complement* $\overline{Q}$ of Q are semi-decidable, where $\overline{Q}$ is the decision problem in which the "yes" instances of Q become "no" instances of $\overline{Q}$, and the "no" instances of Q become "yes" instances of $\overline{Q}$.

Problem 2: During the lectures, we sketched a proof that the problem 1-IN-3-SAT is NP-complete. Fill all the missing details in the proof.

Problem 3: Hilbert's 10th Problem is the following decision problem: Given a polynomial $p(x_1, \ldots, x_k)$ in several variables in which all coefficients are (positive or negative) integers, does this polynomial have a solution consisting of integers?

In 1900, Hilbert challenged mathematicians to come up with an algorithm for solving this problem. In 1971, however, Matiyasevich showed Hilbert's 10th Problem is undecidable.

Consider now the following claim and the proof of it.

Claim: Hilbert's 10th Problem is in NP, hence Hilbert's 10th Problem is decidable.

Proof of Claim: Given a polynomial $p(x_1, \ldots, x_k)$ with integer coefficients, guess integers $a_1, \ldots, a_k$, evaluate $p(a_1, \ldots, a_k)$ and verify that $p(a_1, \ldots, a_k) = 0$. Since evaluation of polynomials can be carried out in polynomial time, it follows that Hilbert's 10th Problem is in NP.

Explain carefully what is wrong with the above "proof".

Problem 4: Give a polynomial-time reduction from 3SAT to the Conjunctive Query Evaluation Problem.

Problem 5: Consider the following decision problem, called NATURAL JOIN NON-EMPTINESS: given relations R_1,...,R_m, is their natural join $R_1 \bowtie \cdots \bowtie R_m$ non-empty? In this version, both the number of relations given and the arity of each relation are arbitrary positive integers.

1. Show that NATURAL JOIN NON-EMPTINESS is in NP.
2. Show that NATURAL JOIN NON-EMPTINESS is NP-hard even if the given relations have arity 3.
3. Let m and k be fixed positive integers. Consider the restriction of NATURAL JOIN NON-EMPTINESS in which we are given at most m relations and each relation has arity at most k. What can you say about the computational complexity of this problem? Is it NP-hard? Is it solvable in polynomial time? Justify your answer.