Philosophical and linguistic applications of team semantics

Class 1: Team Semantics

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Tsinghua Logic Summer School 2024

Practical details

- Time: 9:50 AM-12:15 PM, 8th July-12th July
- Location: Room 3104, Teaching Building No.3, Tsinghua University
- Lecturer: Maria Aloni (University of Amsterdam)
- Guest lecturer: Jialiang Yan (Tsinghua University)
- TAs: Jialiang Yan & Yiqian Wang


Jialiang Yan & Yiqian Wang

Assessment

- 4 sets of homework exercises (at the end of the slides). Only 2 obligatory: today's homework + another of your choice. Deadline: Friday 12 July.
- Final take-home exam distributed on Friday. Deadline: Friday 19 July.

Introduction

Team semantics: formulas interpreted wrt a set of points of evaluation (a **team**) rather than single ones

Evaluation points can be:

- Valuation functions $\mapsto$ propositional team semantics
- First-order assignments $\mapsto$ first-order team semantics
- Possible worlds in a Kripke model $\mapsto$ team-based modal logics

Goal of the course: introduce examples of team-based logics with focus on their philosophical and linguistics applications:

1. BSML: a team-based modal logic
2. Inquisitive semantics: a propositional team semantics
3. Dependence logic: a first-order team semantics

Outline of course

- Day 1. Team semantics: Historical overview & a first introduction to
  - Dependence logic
  - Inquisitive semantics
  - Bilateral state-based modal logic (BSML)
- Day 2. BSML: ignorance and free choice inferences

(1) You may go to Beijing or Shangai $\rightsquigarrow$ You may go to Beijing

- Day 3. qBSML: modified numerals & guest lecture dr. Yan

(2) ?I have at least 2 children.

- Day 4. Inquisitive semantics: questions and inquisitive attitude verbs

(3) J knows whether it is raining & it is not raining $\Rightarrow$ J knows that it is not raining.

- Day 5. Dependence logic: marked indefinites cross-linguistically

(4) Ivan hotel spet' kakuju-nibud' pesniu.
  a. 'Ivan wanted to sing some song [non-specific].'
  b. #'There is a specific song Ivan wanted to sing.'

Team semantics: historical overview

Team semantics: formulas interpreted wrt a set of points of evaluation rather than single ones

屉

(In)Dependence logic tradition

Originally introduced by Hodges (1997) as a compositional semantics for Independence-friendly Logic (Hintikka & Sandu 1989) and later developed further in Dependence Logic (Väänänen 2007), with applications in many areas (database theory, quantum theory, networks, and more);

Semantic/philosophical tradition

Adopted also independently in Inquisitive semantics (Ciardelli & Roelofsen 2011, Ciardelli, Groenendijk & Roelofsen 2018) for characterizing question meanings; and more recently in Bilateral State-Based Modal Logic (MA 2022) to capture non-classical inferences in natural language. Both these logics were strongly influenced by dynamic semantics.

(In)Dependence logic tradition: Henkin Quantifiers

In first-order logic quantifiers are **linearly ordered:**

$$\forall x \exists y \forall z \exists w \phi(x, y, z, w)$$

y depends on x & w depends on both z and x

Leon Henkin
(1921 – 2006)

Henkin (1961) introduced *branching quantifiers*, which can be **partially ordered**:

$$\begin{pmatrix} \forall x & \exists y \\ \forall z & \exists w \end{pmatrix} \phi(x, y, z, w)$$

For every x and z there are y and w, where y *depends only* on x, and w *depends only* on z, such that ϕ.

Or equivalently,

$$\exists f \exists g \forall x \forall z\, \phi(x, f(x), z, g(z))$$

(Enderton, Walkoe, 1970)

First-order logic + Henkin quantifiers $\equiv$ existential second-order logic (ESO)

Independence Friendly (IF) Logic

Hintikka (1974): "It is in fact easy to find any number of examples of branching quantifiers in perfectly grammatical English, including quantifier sentences which do not have any first-order equivalent"

Jaakko Hintikka
(1929 – 2015)

Hintikka's "simple" example:

(5) Some$_y$ relative of each$_x$ villager and some$_w$ relative of each$_z$ townsman hate each other

$$\begin{pmatrix} \forall x & \exists y \\ \forall z & \exists w \end{pmatrix} \phi(x, y, z, w) \qquad \text{(Henkin)}$$

$$(\forall x)(\exists y)(\forall z)(\exists w/\forall x)\phi(x, z, y, w) \qquad \text{(IF)}$$

IF logic (Hintikka & Sandu 1989)

- (in)dependence relations expressed via syntactic scope and the independence indicator '/' (or slash)
- Interpretation in game-theoretic semantics (allows games of imperfect information)

Hodges' semantics

> *There is no realistic hope of formulating compositional truth-conditions for IF first-order sentences, even though I have not given a strict impossibility proof to that effect. (Hintikka 1996)*

Hodges (1997) gives a compositional treatment of IF logic by recursively defining the satisfaction relation in terms of **sets of assignments**, called *trumps*.

Wilfrid Hodges
(born 1941)

Tarski semantics for first-order logic

$M \models_g \phi$, where g is a variable assignment

Hodges' semantics (notation adapted)

$M \models_X \phi$, where X is a set of variable assignments

First-order (FO) logic : $\alpha ::= R(\vec{x}) \mid \neg\alpha \mid \alpha \vee \alpha \mid \alpha \wedge \alpha \mid \exists x\alpha \mid \mid \forall x\alpha$ (with $x \in Var$)

A first-order (FO) model is a pair $M = \langle D, I \rangle$ where domain D is a non-empty set of individuals, and I is an interpretation function for the non-logical part of the vocabulary.

Given $M = \langle D, I \rangle$, a variable assignment is a function $g : V \to D_M$ from a set $V \subseteq Var$ of variables to the domain of D of M.

Dependence Logic

In Dependence Logic (Väänänen 2007), dependence is treated **separately** as an atomic formula

$$\begin{pmatrix} \forall x & \exists y \\ \forall z & \exists w \end{pmatrix} \phi \qquad \text{(Henkin)}$$

$$(\forall x)(\exists y)(\forall z)(\exists w/\forall x)\phi \qquad \text{(IF)}$$

$$\forall x \exists y \forall z \exists w(\phi \wedge dep(z, w)) \qquad \text{(DL)}$$

Jouko Väänänen
(born 1950)

$dep(z, w) \mapsto z$ completely determines w / w functionally depends on z

Dependence atoms

Let M be a FO model and T a set of variable assignments (now called a **team**)

$$M \models_T dep(x, y) \text{ iff for all } i, j \in T : i(x) = j(x) \Rightarrow i(y) = j(y)$$

Illustration

A team is a set of variable assignments

Example of a team (organized collection of data)

T	w (**name**)	x (**age**)	y (**department**)	z (**salary**)
i_1	Aicha	26	maths	2000
i_2	Bo	36	philosophy	4000
i_3	Colin	26	philosophy	2000
i_4	Dew	38	computer science	4000

Dependence atom

$$M \models_T dep(x, y) \text{ iff for all } i, j \in T : i(x) = j(x) \;\Rightarrow\; i(y) = j(y)$$

$dep(x, y) \mapsto x$ completely determines y / y functionally depends on x

True or false?

$dep(y, z)$: department determines salary $\mapsto$ false
$dep(x, z)$: age determines salary $\mapsto$ true
$dep(z, x)$: salary determines age $\mapsto$ false
$dep(w, y)$: name determines department $\mapsto$ true

NEXT: Multiplicity of assignments (a team) necessary to express functional dependence

Multiplicity of assignments necessary to express functional dependence

A singleton team

T_1	x	y	z
i	$\sqrt{2}$	2	1

Does y functionally depend on x? Does z functionally depends on x?
Yes and yes, but trivially so:

$$M \models_{\{i\}} dep(x, y), \text{ for all } i : \{x, y\} \to D_M$$

A multi-membered team

T_2	x	y	z
i_1	$\sqrt{2}$	2	1
i_2	$\sqrt{2}$	2	$\sqrt{2}$
i_3	-2	4	$\sqrt{2}$
i_4	-2	4	2
i_5	$-\sqrt{2}$	2	0

Does y functionally depend on x? Yes, $y = f(x) = x^2$
Does z functionally depends on x? No (e.g., $i_1 \& i_2$)

Dependence Logic: FO + $dep(\vec{x}, \vec{y})$

Language

[Väänänen 2007]

$$\alpha ::= x = x \mid R(\vec{x}) \mid \neg\alpha \mid \alpha \vee \alpha \mid \alpha \wedge \alpha \mid \exists x\alpha \mid \mid \forall x\alpha \quad \text{(with } x \in Var\text{)}$$

$$\phi ::= \alpha \mid \neg\alpha \mid \phi \vee \phi \mid \phi \wedge \phi \mid \exists x\phi \mid \forall x\phi \mid dep(\vec{x}, \vec{x})$$

[NB: $\neg dep(x, y))$ is not a well-formed formula]

Team Semantics

Let $M = \{D, I\}$ be a FO model and T be a team, i.e. a set of assignments $i : V \to D$, with $V \subseteq Var$

$M \models_T \alpha$	$\Leftrightarrow$	for all $j \in T$: $M \models_j \alpha$, when α is a FO formula
$M \models_T \neg\alpha$	$\Leftrightarrow$	for all $j \in T$: $M \not\models_j \alpha$, whenever α is a FO formula
$M \models_T \phi \wedge \psi$	$\Leftrightarrow$	$M \models_T \phi$ and $M \models_T \psi$
$M \models_T \phi \vee \psi$	$\Leftrightarrow$	there are S, S' with $T = S \cup S'$ s.t. $M \models_S \phi$ & $M \models_{S'} \psi$
$M \models_T \forall x\phi$	$\Leftrightarrow$	$M \models_{T[x]} \phi$, where $T[x] = \{i[d/x] : i \in T \text{ and } d \in D_M\}$
$M \models_T \exists x\phi$	$\Leftrightarrow$	there is a function $f : T \to \wp(D_M) \setminus \emptyset$, s.t. $M \models_{T[f/x]} \phi$, where $T[f/x] = \{i[d/x] : i \in T \text{ and } d \in f(i)\}$
$M \models_T dep(\vec{x}, \vec{y})$	$\Leftrightarrow$	for all $i, j \in T$: $i(\vec{x}) = j(\vec{x}) \Rightarrow i(\vec{y}) = j(\vec{y})$

Illustration

$$
\begin{array}{lcl}
M \models_T \alpha & \Leftrightarrow & \text{for all } j \in T: \ M \models_j \alpha, \text{ when } \alpha \text{ is a FO formula} \\
M \models_T \neg\alpha & \Leftrightarrow & \text{for all } j \in T: \ M \not\models_j \alpha, \text{ whenever } \alpha \text{ is a FO formula} \\
M \models_T \phi \vee \psi & \Leftrightarrow & \text{there are } S, S' \text{ with } T = S \cup S' \text{ s.t. } M \models_S \phi \ \& \ M \models_{S'} \psi \\
M \models_T dep(\vec{x}, \vec{y}) & \Leftrightarrow & \text{for all } i, j \in T: i(\vec{x}) = j(\vec{x}) \Rightarrow i(\vec{y}) = j(\vec{y})
\end{array}
$$

Correct or incorrect?

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

$M \models_T x \leq y \mapsto$ yes;

$M \models_T y = z \mapsto$ no; $M \models_T \neg y = z \mapsto$ no;

$M \models_T (\neg y = z \vee y = z) \mapsto$ yes

$M \models_T dep(\emptyset, z) \mapsto$ no

$M \models_T x \leq z \vee dep(\emptyset, z) \mapsto$ yes

Key properties of DL

- Empty Team Property: $M \models_{\emptyset} \phi$, for every ϕ
- Downward closure: $M \models_T \phi$ and $T' \subseteq T \Rightarrow M \models_{T'} \phi$
- Flatness: For every formula α of first-order (FO) logic,

$$M \models_T \alpha \;\Leftrightarrow\; \text{for all } i \in T : M \models_{\{i\}} \alpha$$

- Conservativity: If $\Delta \cup \{\alpha\}$ is a set of FO formulas, then

$$\Delta \models_{team} \alpha \;\Leftrightarrow\; \Delta \models_{classical} \alpha$$

But: Flatness and conservativity do not hold in general:

- $dep(x, y)$ is not flat, since $M \models_{\{i\}} dep(x, y)$ always holds, whereas $M \not\models_T dep(x, y)$ for some team T.

Entailment and equivalence

- For a set Δ of formulas, we write $M \vDash_T \Delta$, if $M \vDash_T \phi$ for all $\phi \in \Delta$
- $\Delta \vDash \psi$ (Δ *entails* ψ), if $M \vDash_T \Delta$ implies $M \vDash_T \psi$, for all models M and teams T on M with $\mathrm{Fv}(\Delta \cup \{\psi\}) \subseteq dom(T)$.
- We also write simply $\phi_1, \ldots, \phi_n \vDash \psi$ for $\{\phi_1, \ldots, \phi_n\} \vDash \psi$.
- If both $\phi \vDash \psi$ and $\psi \vDash \phi$, then we write $\phi \equiv \psi$ and say that ϕ and ψ are *equivalent*.

(In)Dependence logic tradition: summary historical overview

Branching quantifiers (Henkin 1961)

$$\begin{pmatrix} \forall x & \exists y \\ \forall z & \exists w \end{pmatrix} \phi$$

Independence-Friendly Logic (Hintikka and Sandu, 1989)

$$(\forall x)(\exists y)(\forall z)(\exists w/\forall x)\phi$$

Dependence Logic (Väänänen 2007)

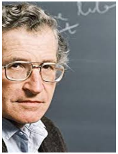
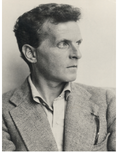
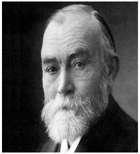
$$\forall x \exists y \forall z \exists w(\phi \wedge dep(z, w))$$

(Enderton, Walkoe, 1970, Vaananen 2007)

FO + Henkin quantifiers $\equiv$ ESO $\equiv$ IF-Logic $\equiv$ Dependence Logic

The semantic/philosophical tradition

- **Formal semantics** uses logic to analyze linguistic meaning
  - Roots in philosophy of language (Frege, Russell, Wittgenstein) and theoretical linguistics (Chomsky);
  - Richard Montague (1930–1971, mathematician and philosopher) pioneered a logical approach to natural language meaning;
  - Barbara H. Partee (born 1940, linguist and philosopher) established formal semantics as independent (interdisciplinary) field of research.

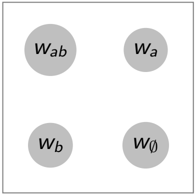
- Indirect method: semantic datas explained via translation

$$\text{Natural Language} \mapsto \text{Logical Language} \Rightarrow \text{Models}$$

- Many challenges: (i) How to collect semantic datas in a reliable way? (ii) How to translate natural language expressions into a logical language in a systematic and compositional way? But also (iii) which logic?

Intensional semantics and possible worlds

- Intensional semantics [Montague 1973]
  In intensional semantics, a model $M = (W, \dots)$ comes with a universe W of possible worlds, representing different states of affairs



Logical space

- Truth conditions & semantic content
  The semantics of a sentence is given by specifying at which worlds it is true:

$$M, w \models \phi, \text{ where } w \in W$$

  Sentences express propositions characterised as sets of possible worlds

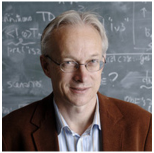
$$|\phi|_M = \{w \in W_M \mid M, w \models \phi\}$$

Dynamic Semantics for Natural Language

Core ideas due independently to Kamp (1981) and Heim (1982)


Developed in the 1990's by Groenendijk & Stokhof, Veltman, van Benthem, and more



Static vs dynamic view on meaning

- Static view: meanings are truth conditions
  - To understand the meaning of a sentence is to understand what has to be the case for the sentence to be true
- In static semantics, sentences are true or false wrt to worlds in models, and express propositions characterised as sets of possible worlds:

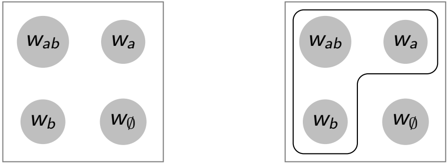
$$|\phi|^C_M = \{w \in W_M \mid M, w \models \phi\}$$

- Dynamic view: meanings are context change potentials
  - The meaning of any linguistic expression is dependent on the context of interpretation
  - The meaning of any linguistic expression is its ability to change the context of interpretation
- Contexts $\mapsto$ information states of language users
- In dynamic semantics, sentences denote relations over information states:

$$s_0|\phi|^{Dyn}_M \quad \begin{matrix} \nearrow & s_1 \\ \rightarrow & s_2 \\ \searrow & s_3 \end{matrix}$$

Information states

- In propositional systems, info states are modeled as sets $s \subseteq W$ of possible worlds ($w \in s \Leftrightarrow w$ is compatible with the information available in s)



(a) Logical space (b) Info state

- In first-order systems, info states are sets of world-assignment pairs.
- Thus dynamic semantics example of a team-based system $\mapsto$ formulas are evaluated with respect to sets of evaluation points

Inquisitive semantics & BSML

- Inquisitive Semantics, building on this tradition, evaluates formulas with respect to information states defined as sets of possible worlds:

$$M, s \models \phi, \text{ where } s \subseteq W$$

- Bilateral State-based Modal Logic (BSML) combines insights from both dependence logic and dynamic-inquisitive traditions
  - Formulas evaluated wrt sets of possible worlds, representing information states as in dynamic/inquisitive tradition
  - Disjunction interpreted as in Dependence Logic
  - Modals interpreted as in Inquisitive Modal Logic
  - ...
- Let's have a closer look!

Inquisitive Semantics

Goal: uniform semantics for declarative and interrogative sentences

Core idea:

- Interrogatives introduce a number of propositional alternatives;
- Declaratives just introduce one alternative.

Jeroen Groenendijk
(1949-2023)

Question meanings modeled using inquisitive disjunction (Ciardelli & Roelofsen 2011; Ciardelli, Groenendijk & Roelofsen 2018)

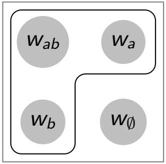
Inquisitive disjunction

Let $M = (W, V)$ be an intensional model and $s \subseteq W$ a *state*.

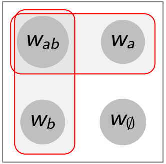
$$M, s \models \phi \mathbin{\vee\!\!\!/} \psi \Leftrightarrow M, s \models \phi \text{ or } M, s \models \psi$$

Classical vs inquisitive view on meaning

- Classical semantics
  - Formula interpreted wrt possible worlds in a model $M = (W, V)$:
    $$M, w \models \phi, \text{ where } w \in W$$
  - Semantic content $\mapsto$ set of possible worlds (classical proposition)
    $$|\phi|^C_M = \{w \in W \mid M, w \models \phi\}$$
- Inquisitive semantics
  - Formula interpreted wrt states (sets of possible worlds) in M:
    $$M, s \models \phi, \text{ where } s \subseteq W$$
  - Semantic content $\mapsto$ set of states (inq-proposition)
    $$|\phi|^I_M = \{s \subseteq W \mid M, s \models \phi\}$$



(c) classical: $|a \vee b|^C$



(d) inquis: $|a \mathbin{\vee\!\!\!/} b|^I$

Core idea inquisitive semantics

Questions as inquisitive disjunctions

$$?\phi := \phi \vee\!\!\!\!/ \;\neg\phi$$

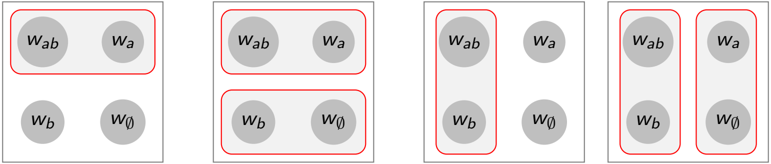
(6) Aicha is coming $\mapsto a$ $\qquad |a|^I = \{\{w_a, w_{ab}\}, \{w_a\}, \{w_{ab}\}, \emptyset\}$

(7) Is Aicha coming? $\mapsto a \vee\!\!\!\!/ \;\neg a$ $\qquad |a \vee\!\!\!\!/ \;\neg a|^I = \{\{w_a, w_{ab}\}, \{w_a\}, \{w_{ab}\}, \{w_b, w_\emptyset\}, \{w_b\}, \{w_\emptyset\}, \emptyset\}$

(8) Bo is coming $\mapsto$ b $\qquad |b|^I = \{\{w_b, w_{ab}\}, \{w_b\}\{w_{ab}\}, \emptyset\}$

(9) Is Bo coming? $\mapsto b \vee\!\!\!\!/ \;\neg b$ $\qquad |b \vee\!\!\!\!/ \;\neg b|^I = \{\{w_b, w_{ab}\}, \{w_b\}, \{w_{ab}\}, \{w_a, w_\emptyset\}, \{w_a\}, \{w_\emptyset\}, \emptyset\}$

Maximal elements in an inq-proposition $|\phi|^I$ are called alternatives



(e) $|a|^I$ (f) $|?a|^I$ (g) $|b|^I$ (h) $|b?|^I$

Inquisitive Logic: CPL + $\vee\!\!\!/$

Language

[Ciardelli & Roelofsen 2011]

$$\phi := p \mid \bot \mid \phi \to \phi \mid \phi \wedge \phi \mid \phi \mathbin{\vee\!\!\!/} \phi \qquad \text{with } p \in PROP$$

Defined connectives: $\neg\phi := \phi \to \bot$ & $\phi \vee_* \psi := \neg(\neg\phi \wedge \neg\psi)$

Team/state-based semantics

Given a model $M = (W, V)$ and an information state $s \subseteq W$,

- $M, s \models p \Leftrightarrow$ for all $w \in s : V(p, w) = 1$
- $M, s \models \bot \Leftrightarrow s = \emptyset$
- $M, s \models \phi \wedge \psi \Leftrightarrow M, s \models \phi$ and $M, s \models \psi$
- $M, s \models \phi \to \psi \Leftrightarrow$ for all $t \subseteq s : t \models \phi$ implies $t \models \psi$
- $M, s \models \phi \mathbin{\vee\!\!\!/} \psi \Leftrightarrow M, s \models \phi$ or $M, s \models \psi$

Defined connectives

- $M, s \models \neg\phi \Leftrightarrow$ for all $t \subseteq s : t \not\models \phi$, unless $t = \emptyset$
- $M, s \models \phi \vee_* \psi \Leftrightarrow$ for all $w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$

Key properties of IL

- Empty Team Property: $M, \emptyset \models \phi$, for every ϕ
- Downward closure: $M, s \models \phi$ and $t \subseteq s \Rightarrow M, t \models \phi$
- Flatness: For every formula α of propositional logic,

$$M, s \models \alpha \;\Leftrightarrow\; \text{for all } w \in s : M, \{w\} \models \alpha$$

- Conservativity: If $\Delta \cup \{\alpha\}$ is a set of propositional logic formulas, then

$$\Delta \models_{inquisitive} \alpha \;\Leftrightarrow\; \Delta \models_{classical} \alpha$$

- **But**: $\phi \mathbin{\vee\!\!\!/} \psi$ is not flat, since $M, \{w\} \models p \mathbin{\vee\!\!\!/} \neg p$ always holds, whereas $M, s \not\models p \mathbin{\vee\!\!\!/} \neg p$ for some states, e.g. $s = \{w_p, w_\emptyset\}$.

Dependence Logic and Inquisitive Semantics

Unexpected similarities between Dependence Logic and Inquisitive Semantics, the two logics satisfy the same key properties.

In Propositional Inquisitive Semantics and in Propositional Dependence Logic, meanings are the same kind of object, namely downward closed sets of states/teams, defined as sets of propositional valuations (Yang 2014)

Fan Yang

Propositional dependence logic: CPL + $dep(\vec{p}, \vec{p})$

Language [Yang & Väänänen 17]

$$\phi := p \mid \neg p \mid \phi \wedge \phi \mid \phi \vee \phi \mid dep(\vec{p}, \vec{p}) \qquad \text{with } p \in PROP$$

Team semantics Let $t \subseteq 2^{PROP}$ be a team/state, i.e. a set of valuations.

- $t \models p \Leftrightarrow$ for all $v \in t : v(p) = 1$
- $t \models \neg p \Leftrightarrow$ for all $v \in t : v(p) = 0$
- $t \models \phi \wedge \psi \Leftrightarrow M, t \models \phi$ and $M, t \models \psi$
- $t \models \phi \vee \psi \Leftrightarrow t = t_1 \cup t_2$ s.t. $M, t_1 \models \phi$ and $M, t_2 \models \psi$
- $t \models dep(\vec{p}, \vec{q}) \Leftrightarrow$ for all $i, j \in t : i(\vec{p}) = j(\vec{p}) \Rightarrow i(\vec{q}) = j(\vec{q})$

Constancy atom: $t \models dep(\vec{p})$ iff for all $v, u \in t : v(\vec{p}) = u(\vec{p})$

- Fact 1: $\models dep(p) \vee dep(p)$
- Fact 2: $dep(p) \vee dep(p) \not\models dep(p)$
- Fact 3: $dep(p) \equiv p \mathbin{\vee\!\!\!/} \neg p \equiv ?p$ (inquisitive semantics)

Yang & Väänänen (2017) also discuss non-emptiness atom $\textsc{ne}$

- $t \models \textsc{ne} \Leftrightarrow t \neq \emptyset$

Bilateral State-Based Modal Logic (BSML)

- Classical modal logic: [Kripke model $M = \langle W, R, V \rangle$]

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

- BSML: team-based and bilateral

$$M, s \models \phi, \text{ “}\phi\text{ is assertable in } s\text{”}, \quad \text{with } s \subseteq W$$
$$M, s \dashv \phi, \text{ “}\phi\text{ is rejectable in } s\text{”}, \quad \text{with } s \subseteq W$$

- Teams $\mapsto$ information states, as in inquisitive semantics
- NE only source of non-classicality
- Disjunction interpreted as in Dependence Logic
- Modals interpreted as in Inquisitive Modal Logic

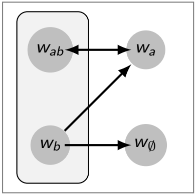
BSML: Classical Modal Logic + NE

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE} \qquad \text{with } p \in A$$

Bilateral team semantics

Given a Kripke model $M = \langle W, R, V \rangle$ & $s, t, t' \subseteq W$



$M, s \models p$	iff	for all $w \in s : V(w, p) = 1$
$M, s \mathrel{=\!\!\mid} p$	iff	for all $w \in s : V(w, p) = 0$
$M, s \models \neg\phi$	iff	$M, s \mathrel{=\!\!\mid} \phi$
$M, s \mathrel{=\!\!\mid} \neg\phi$	iff	$M, s \models \phi$
$M, s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$
$M, s \mathrel{=\!\!\mid} \phi \vee \psi$	iff	$M, s \mathrel{=\!\!\mid} \phi$ & $M, s \mathrel{=\!\!\mid} \psi$
$M, s \models \phi \wedge \psi$	iff	$M, s \models \phi$ & $M, s \models \psi$
$M, s \mathrel{=\!\!\mid} \phi \wedge \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \mathrel{=\!\!\mid} \phi$ & $M, t' \mathrel{=\!\!\mid} \psi$
$M, s \models \Diamond\phi$	iff	for all $w \in s : \exists t \subseteq R[w] : t \neq \emptyset$ & $M, t \models \phi$
$M, s \mathrel{=\!\!\mid} \Diamond\phi$	iff	for all $w \in s : M, R[w] \mathrel{=\!\!\mid} \phi$
$M, s \models \mathrm{NE}$	iff	$s \neq \emptyset$
$M, s \mathrel{=\!\!\mid} \mathrm{NE}$	iff	$s = \emptyset$ [where $R[w] = \{v \in W \mid wRv\}$]

Logical Validity. $\phi \models \psi$ iff for all M, s: $M, s \models \phi \Rightarrow M, s \models \psi$

Proof Theory. See Anttila 2021; Anttila et al. 2024.

Key properties of BSML

Closure properties: definitions

- ϕ has the *empty state property*: $M, \emptyset \vDash \phi$ for all M;
- ϕ is *downward closed*: $[M, s \vDash \phi \text{ and } t \subseteq s] \implies M, t \vDash \phi$;
- ϕ is *union closed*: $[M, s \vDash \phi \text{ for all } s \in S \neq \emptyset] \implies M, \bigcup S \vDash \phi$;
- ϕ is *flat*, provided $M, s \vDash \phi \iff M, \{w\} \vDash \phi$ for all $w \in s$.

Facts

flatness $\Leftrightarrow$ downward closure, union closure & empty state property

Union Closure: ϕ is union closed, for all ϕ in BSML

Flatness: α is flat, for all NE-free formula α in BSML

But: formulas with NE are not flat, they may lack downward closure and the empty state property;

- $M, \emptyset \not\models \mathrm{NE}$ (failure of empty team property)
- $M, \{w_a, w_b\} \models (a \vee (b \wedge \mathrm{NE}))$, but $M, \{w_a\} \not\models (a \vee (b \wedge \mathrm{NE}))$ (failure of downward closure)

Key properties of BSML radically different from those of Dependence Logic & Inquisitive Logic.

Summary and outlook

Team semantics: formulas interpreted in teams rather than single points of evaluations

1. Bilateral State-based Modal Logic: CML + non-emptiness atom NE
   - Teams: sets of possible worlds in a Kripke model
   - Properties: downward-closure^{no}, empty team property^{no}, union closure^{yes}
   - Extensions & applications: in days 2 and 3
2. Inquisitive Logic: CPL + inquisitive disjunction $\mathbin{\backslash\!\!\!\vee}$
   - Teams: sets of possible worlds (or equivalently propositional valuations)
   - Properties: downward-closure^{yes}, empty team property^{yes}, union closure^{no}
   - Extensions & applications: in day 4
3. Dependence Logic: FO + dependence atoms $dep(x, y)$
   - Teams: sets of first order assignments
   - Properties: downward-closure^{yes}, empty team property^{yes}, union closure^{no}
   - Extensions & applications: in day 5

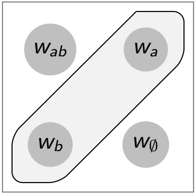
Three notions of disjunction

$s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $t \models \phi$ & $t' \models \psi$	[split]
$s \models \phi \mathbin{\vee\!\!\!/} \psi$	iff	$s \models \phi$ or $s \models \psi$	[inquisitive]
$s \models \phi \vee_* \psi$	iff	for all $w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$	[flat]

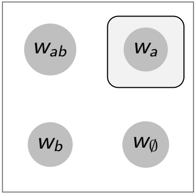
Some facts

Let α, β be flat

- $\alpha \vee \beta \equiv \alpha \vee_* \beta$ (**but**: $\alpha \vee (b \wedge \mathrm{NE}) \not\equiv \alpha \vee_* (b \wedge \mathrm{NE})$)
- $\alpha \mathbin{\vee\!\!\!/} \beta \models \alpha \vee_{(*)} \beta$, but $\alpha \vee_{(*)} \beta \not\models \alpha \mathbin{\vee\!\!\!/} \beta$



(i) $\models a \vee_{(*)} b$, $\not\models a \mathbin{\vee\!\!\!/} b$



(j) $\models a \vee_{(*)} b$, $\models a \mathbin{\vee\!\!\!/} b$

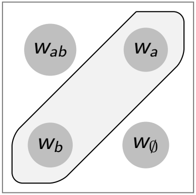
Appendix: Three notions of disjunction

$s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $t \models \phi$ & $t' \models \psi$	[split]
$s \models \phi \mathbin{\vee\!\!\!/} \psi$	iff	$s \models \phi$ or $s \models \psi$	[inquisitive]
$s \models \phi \vee_* \psi$	iff	for all $w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$	[flat]

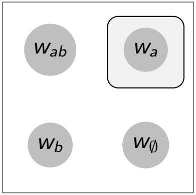
Flatness and LEM

Let α, β be flat

- $\alpha \vee \beta$ flat
- $\alpha \mathbin{\vee\!\!\!/} \beta$ not flat ($\{i\} \models \alpha \mathbin{\vee\!\!\!/} \neg\alpha$ always, but $s \not\models \alpha \mathbin{\vee\!\!\!/} \neg\alpha$, for some s)
- $\not\models \alpha \mathbin{\vee\!\!\!/} \neg\alpha$ (LEM fails in inquisitive logic)
- $\models \alpha \vee \neg\alpha$ (LEM holds even if bivalence fails)



(k) $\models a \vee \neg a$, $\not\models a \mathbin{\vee\!\!\!/} \neg a$



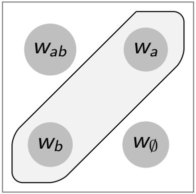
(l) $\models a \vee \neg a$, $\models a \mathbin{\vee\!\!\!/} \neg a$

Three notions of disjunction

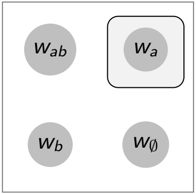
$s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $t \models \phi$ & $t' \models \psi$	[split]
$s \models \phi \mathbin{\vee\!\!\!/} \psi$	iff	$s \models \phi$ or $s \models \psi$	[inquisitive]
$s \models \phi \vee_* \psi$	iff	for all $w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$	[flat]

Different conceptualisations for different notions

- $s \models \alpha \vee \beta \rightsquigarrow$ "agent in state s has enough evidence to assert $\alpha \vee \beta$"
- $s \models \alpha \mathbin{\vee\!\!\!/} \beta \rightsquigarrow$ "s contains enough info to resolve whether α is true or β"



(m) $\models (a \vee b)$, $\not\models (a \mathbin{\vee\!\!\!/} b)$



(n) $\models (a \vee b)$, $\models (a \mathbin{\vee\!\!\!/} b)$

- $\vee$ allows a direct account of indeterminacy of disjunction

(10) A: X or Y will be elected. (Grice 1991, p 82)
B: That's not so; X or Y or Z will be elected.

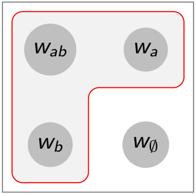
Three notions of disjunction

$s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $t \models \phi$ & $t' \models \psi$	[split]
$s \models \phi \mathbin{\vee\!\!\!/} \psi$	iff	$s \models \phi$ or $s \models \psi$	[inquisitive]
$s \models \phi \vee_* \psi$	iff	for all $w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$	[flat]

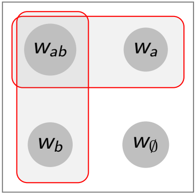
Different semantic contents generated by different notions

Let α, β be flat and logically independent.

1. $\{s \mid s \models (\alpha \mathbin{\vee\!\!\!/} \beta)\}$ is inquisitive, i.e. it contains more than one maximal state, aka alternative;
2. $\{s \mid s \models (\alpha \vee \beta)\}$ is not inquisitive.



(o) not inquistive: $a \vee b$



(p) inquisitive: $a \mathbin{\vee\!\!\!/} b$

- Alternatives generated by $\mathbin{\vee\!\!\!/}$ largely used in semantics for question meanings and more

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

   1. $M \models_T dep(x, y)$
   2. $M \models_T dep(x, v)$
   3. $M \models_T dep(v, x)$
   4. $M \models_T dep(xy, z)$
   5. $M \models_T dep(\varnothing, v)$

2. Determine whether the following claims are correct given T as depicted.

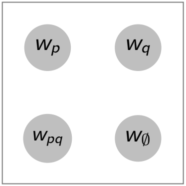
T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

   1. $M \models_T x \leq z$
   2. $M \models_T x \leq z \vee z \leq y$
   3. $M \models_T \exists v \; v \leq z$
   4. $M \models_T \exists v (v \leq z \wedge y \leq v)$
   5. $M \models_T \forall v \; z \leq v$

3. Show that $dep(x, y) \vee dep(x, y) \not\models dep(x, y)$

Exercises (Inquisitive logic)

1. Consider the following intensional model:



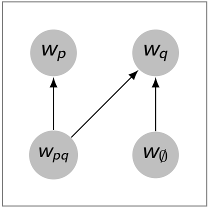
   Determine whether the following statements are correct:

   1. $\{w_p, w_{pq}\} \models p$
   2. $\{w_p, w_{pq}\} \models q$
   3. $\{w_p, w_{pq}\} \models \neg q$
   4. $\{w_p\} \models p \mathbin{\vee\!\!\!/} q$
   5. $\{w_p, w_q\} \models p \mathbin{\vee\!\!\!/} q$
   6. $\{w_p, w_{pq}\} \models p \rightarrow q$

2. Show that Inquisitive Logic invalidates LEM: $\not\models \phi \mathbin{\vee\!\!\!/} \neg\phi$

Exercise (BSML)

1. Consider the following Kripke model:



Determine whether the following statements are correct:

1. $\{w_p\} \models p \vee q$
2. $\{w_p\} \models p \vee (q \wedge \text{NE})$
3. $\{w_p, w_q\} \models p \vee q$
4. $\{w_p, w_q\} \models (p \wedge \text{NE}) \vee (q \wedge \text{NE})$
5. $\{w_\emptyset\} \models \Diamond(p \vee q)$
6. $\{w_\emptyset\} \models \Diamond((p \wedge \text{NE}) \vee (q \wedge \text{NE}))$
7. $\{w_{pq}\} \models \Diamond((p \wedge \text{NE}) \vee (q \wedge \text{NE}))$

2. Does BSML validate LEM $\phi \vee \neg\phi$?

Exercises (bonus)

1. Show that inquisitive logic and dependence logic are not union closed
2. Show that BSML is union closed
3. Consider the following new closure property

   Convexity
   ϕ is convex if $[s \models \phi, t \models \phi$ and $s \subseteq u \subseteq t]$ implies $u \models \phi$

   Check whether convexity holds for all formulas in Dependence Logic, Inquisitive Logic and BSML.