Philosophical and linguistic applications of team semantics

Class 2: BSML

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Outline of course

- Day 1. Team semantics: Historical overview & a first introduction to
  - Dependence logic
  - Inquisitive semantics
  - Bilateral state-based modal logic (BSML)
- Day 2. **BSML: ignorance and free choice inferences** ⇐
- Day 3. qBSML: modified numerals & guest lecture dr. Yan
- Day 4. Inquisitive semantics: questions and inquisitive attitude verbs
- Day 5. Dependence logic: marked indefinites cross-linguistically

Plan of today

- N∅thing is logical & neglect-zero hypothesis
- Neglect-zero in BSML
- Ignorance & the acquisition of disjunction
- Free choice
- $BSML^+$ vs $BSML^*$ vs $BSML^\emptyset$

Literature

- MA 2022. "Logic and conversation: the case of free choice." *Semantics and Pragmatics* (2022), 15(5);
- MA, Aleksi Anttila & Fan Yang. "State-based Modal Logics for Free Choice". To appear in *Notre Dame Journal of Formal Logic*.

N∅thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments (Grice)
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency as crucial pragmatic/cognitive factor
- Main conclusion: deviations from classical logic consequence of pragmatic enrichments albeit not of the canonical Gricean kind

Non-classical inferences

Free choice (FC)

(1) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(2) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

(4) The prize is in the attic *or* in the garden $\rightsquigarrow$ speaker doesn't know where
(5) ? I have two *or* three children. [Grice 1989]

- In the standard approach, ignorance inferences are conversational implicatures [Sauerland 2004, Soames 1989, Horn 1989]
- Less consensus on FC inferences analysed as conversational implicatures; grammatical implicatures; semantic entailments; . . .

Novel hypothesis: neglect-zero

- FC and ignorance inferences are [$\neq$ semantic entailments]
  - Not the result of Gricean reasoning [$\neq$ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [$\neq$ scalar implicatures]
- But rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers create structures representing reality[1] and in doing so they systematically neglect structures which verify the sentence by virtue of an empty configuration (*zero-models*)

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- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets [Nieder 2016, Bott et al, 2019]

[1] Johnson-Laird (1983) *Mental Models*. Cambridge University Press.

Novel hypothesis: neglect-zero

Illustrations

(6) Every square is black.

- a. Verifier: [■, ■, ■]
- b. Falsifier: [■, □, ■]
- c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]

(7) Less than three squares are black.

- a. Verifier: [■, □, ■]
- b. Falsifier: [■, ■, ■]
- c. Zero-models: [ ]; [□, □, □]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]

- Cognitive difficulty of zero-models confirmed by experimental findings from number cognition and has been argued to explain
  - the special status of 0 among the natural numbers [Nieder, 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott et al., 2019]
  - existential import & connexive principles from Aristotle (*every A is B* ⇒ *some A is B*; *not (if A then not A)*) [MA & Knudstorp 2024]
- Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and related inferences

Novel hypothesis: neglect-zero

Illustrations

(8) It is raining.

- a. Verifier: [//// //// ////]
- b. Falsifier: [☼☼☼]
- c. Zero-models: none

(9) It is snowing.

- a. Verifier: [❄❄❄]
- b. Falsifier: [☼☼☼]; [//// //// ////]; . . .
- c. Zero-models: none

(10) It is raining or snowing.

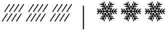
- a. Verifier: [//// //// //// | ❄❄❄]
- b. Falsifier: [☼☼☼]
- c. **Zero-models**: [//// //// ////]; [❄❄❄]

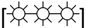
- Two models in (10-c) are **zero-models** because they verify the sentence by virtue of an empty witness for one of the disjuncts
- Ignorance effects arise because such zero-models are cognitively taxing and therefore disregarded

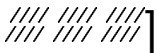
A new conjecture: no-split

A closer look at the disjunctive case

(11) It is raining or snowing.

a. Verifier: []   [⇐ "split" state]

b. Falsifier: []

c. Zero-models: []; []

- The "split" verifier in (11-a) involves the entertainment of two alternatives $\mapsto$ also a cognitively difficult operation

  NO-SPLIT CONJECTURE   [Klochowicz, Sbardolini & MA 2024]
  the ability to split states (entertain multiple alternatives) is acquired late

- The combination of neglect-zero & no-split can explain non-classical inferences observed in pre-school children

No-split and the acquisition of 'or'

- Basic data: pre-school children interpret *or* as *and* [e.g., Singh *et al* 2016, Cochard 2023, Bleotu *et al* 2024]:

(12) The boy is holding an apple or a banana = The boy is holding an apple and a banana $\alpha \vee \beta = \alpha \wedge \beta$

(13) Every boy is holding an apple or a banana = Every boy is holding an apple and a banana $\forall x(\alpha \vee \beta) = \forall x(\alpha \wedge \beta)$

(14) Liz can buy a croissant or a donut = Liz can buy a croissant and a donut $\Diamond(\alpha \vee \beta) = \Diamond(\alpha \wedge \beta)$

(15) The boy is not holding an apple or a banana = The boy is neither holding an apple nor a banana $\neg(\alpha \vee \beta) = \neg\alpha \wedge \neg\beta$

- Two different explanations:
  - Singh et al: children can compute scalar implicatures and can exhaustify alternatives, but don't have access to lexical alternatives. So derive $\alpha \wedge \beta$ from $\alpha \vee \beta$ as a scalar implicature using exh-ALT $= \{\alpha \wedge \neg\beta, \beta \wedge \neg\alpha\}$
  - Nihil: beside neglecting zero-models, children further lack the ability to split states, i.e. cannot engage with alternative epistemic possibilities, cannot picture different ways the world might be.

BSML: teams and bilateralism

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]

Classical vs team-based modal logic

$[M = \langle W, R, V \rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic: (support in teams)

$$M, t \models \phi, \text{ where } t \subseteq W$$

Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states [Dekker93; Groenendijk$^+$96; Ciardelli$^+$18]
- Assertion & rejection conditions modelled rather than truth

$$M, s \models \phi, \text{ “}\phi \text{ is assertable in } s\text{”}, \quad \text{with } s \subseteq W$$
$$M, s \mathrel{=\!\!|} \phi, \text{ “}\phi \text{ is rejectable in } s\text{”}, \quad \text{with } s \subseteq W$$

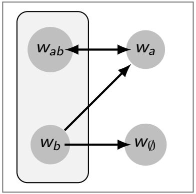
- Split disjunction $\vee$ rather than inquisitive disjunction $\mathbin{\vee\!\!\!\vee}$
- Neglect-zero tendency modelled by NE [Yang & Väänänen 2017]
- BSML$^{\mathrm{F}}$: No-split modelled via a flattening operator F

BSML: Classical Modal Logic + NE

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE} \qquad \text{with } p \in A$$

Bilateral team semantics



Given a Kripke model $M = \langle W, R, V\rangle$ & states $s, t, t' \subseteq W$

$$\begin{array}{rll}
M, s \models p & \text{iff} & \text{for all } w \in s : V(w,p) = 1 \\
M, s \dashv p & \text{iff} & \text{for all } w \in s : V(w,p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \vee \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi \\
M, s \dashv \phi \vee \psi & \text{iff} & M, s \dashv \phi \;\&\; M, s \dashv \psi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \;\&\; M, s \models \psi \\
M, s \dashv \phi \wedge \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \;\&\; M, t \dashv \phi \;\&\; M, t' \dashv \psi \\
M, s \models \Diamond\phi & \text{iff} & \text{for all } w \in s : \exists t \subseteq R[w] : t \neq \emptyset \;\&\; M, t \models \phi \\
M, s \dashv \Diamond\phi & \text{iff} & \text{for all } w \in s : M, R[w] \dashv \phi \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\
M, s \dashv \mathrm{NE} & \text{iff} & s = \emptyset \qquad\qquad [\text{where } R[w] = \{v \in W \mid wRv\}]
\end{array}$$

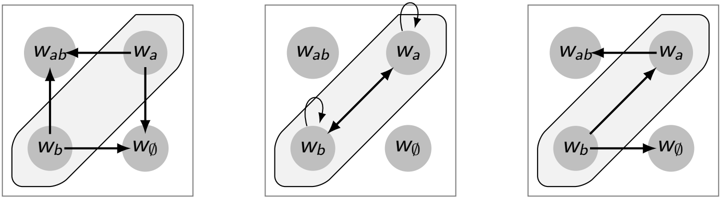
Are the following formulas supported? a (no); b (yes); $\neg a$ (no); $a \vee \neg a$ (yes); $b \vee \neg b$ (yes);$\Diamond a$ (yes); $\Diamond b$ (no); $\neg\Diamond b$ (yes).

Logical Consequence. $\phi \models \psi$ iff for all M, s: $M, s \models \phi \Rightarrow M, s \models \psi$

Proof Theory. See Anttila 2021; Anttila et al. 2024.

Team-sensitive constraints on accessibility relation

- R is indisputable in (M, s) iff $\forall w, v \in s : R[w] = R[v]$
  $\mapsto$ all worlds in s access exactly the same set of worlds
- R is state-based in (M, s) iff $\forall w \in s : R[w] = s$
  $\mapsto$ all and only worlds in s are accessible within s



(a) indisputable

(b) state-base (& indisputable)

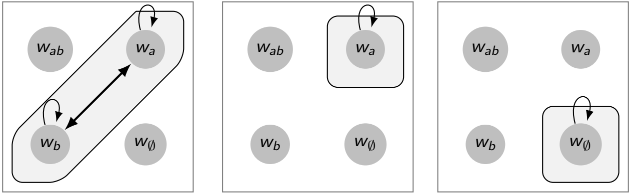
(c) neither

Deontic vs epistemic modals

- Difference deontic vs epistemic modals captured by different properties of accessibility relation:
  - Epistemics: R is state-based
  - Deontics: R is possibly indisputable (e.g. in performative uses)

Neglect-zero effects in BSML: split disjunction

- A state s supports a **disjunction** $(\phi \vee \psi)$ iff s is the union of two substates, each supporting one of the disjuncts



(d) Verifier (e) **Zero-model** (f) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\}$ verifies $(a \vee b)$ by virtue of an empty witness for the second disjunct, $\{w_a\} = \{w_a\} \cup \emptyset$ & $M, \emptyset \models b$ [$\mapsto$ **zero-model**]
- Main idea: define neglect-zero enrichments, $[\ ]^+$, whose core effect is to rule out such zero-models
- Implementation: $[\ ]^+$ defined using NE ($s \models \text{NE}$ iff $s \neq \emptyset$), which models neglect-zero in the logic

BSML: neglect-zero enrichment

Non-emptiness

NE is supported in a state if and only if the state is not empty

$$
\begin{array}{lll}
M, s \models \text{NE} & \text{iff} & s \neq \emptyset \\
M, s \mathrel{=}\!\!| \text{NE} & \text{iff} & s = \emptyset
\end{array}
$$

Neglect-zero enrichment

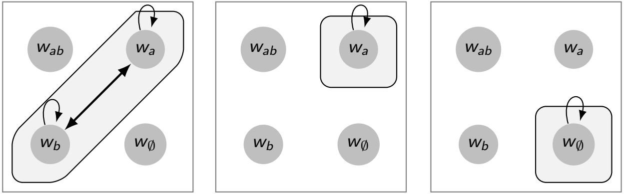
For NE-free α, $[\alpha]^+$ defined as follows:

$$
\begin{array}{rcl}
[p]^+ & = & p \wedge \text{NE} \\
[\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \text{NE} \\
[\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \text{NE} \\
[\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \text{NE} \\
[\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \text{NE}
\end{array}
$$

$[\ ]^+$ enriches formulas with the requirement to satisfy NE distributed along each of their subformulas

Neglect-zero effects in BSML: enriched disjunction

- *s* supports an **enriched disjunction** $[\phi \vee \psi]^+$ iff *s* is the union of two non-empty substates, each supporting one of the disjuncts



(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $=\!| \; [a \vee b]^+$

- An enriched disjunction requires both disjuncts to be live possibilities

(16) It is raining or snowing $\rightsquigarrow$ It might be raining and it might be snowing
$[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (where *R* is state-based)

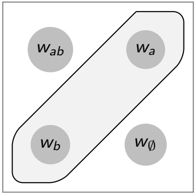
- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to *positive* disjunctions[2]
  - $\mapsto$ we derive FC and related effects (for enriched formulas);
  - $\mapsto$ $[\ ]^+$-enrichment vacuous under single negation.

[2] MA (2022) Logic and Conversation: the case of free choice. *Semantics and Pragmatics* 15(5).

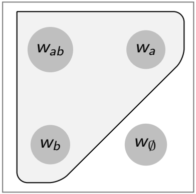
Zero and no-zero models

(M, s) is a zero-model for α iff $M, s \models \alpha$, but $M, s \not\models [\alpha]^+$
(M, s) is a no-zero verifier for α iff $M, s \models [\alpha]^+$

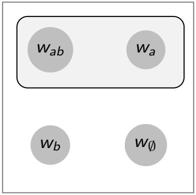
Many no-zero verifiers for enriched disjunction



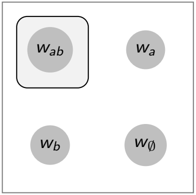
(d) no-zero & scalar $\models \neg(a \wedge b)$



(e) no-zero, non-scalar $\not\models \neg(a \wedge b)$

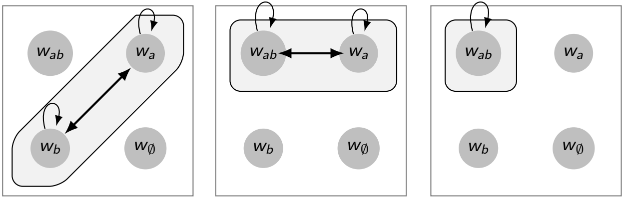


(f) no-zero, non-scalar & no-uncertain $\not\models \neg\Box_e a$



(g) no-zero, non-scalar, no-uncertain & no-split $\models (a \wedge b)$

Neglect-zero effects in BSML: possibility vs uncertainty



(a) scalar (b) no-uncertain (c) no-split

Figure: Models for enriched $[a \vee b]^+$.

- Two components of full ignorance ('speaker doesn't know which'):[3]

  (17) It is raining or it is snowing ($\alpha \vee \beta$) $\rightsquigarrow$

  a. Uncertainty: $\neg\Box_e\alpha \wedge \neg\Box_e\beta$

  b. Possibility: $\Diamond_e\alpha \wedge \Diamond_e\beta$ (equiv $\neg\Box_e\neg\alpha \wedge \neg\Box_e\neg\beta$)

- **Fact:** Only possibility derived as neglect-zero effect:
  - $[a \vee b]^+ \models \Diamond_e a \wedge \Diamond_e b$ (if R is state-based)
  - $\{w_{ab}, w_a\} \models [a \vee b]^+$, but $\not\models \neg\Box_e a$
  - $\{w_{ab}\} \models [a \vee b]^+$, but $\not\models \neg\Box_e a$; $\not\models \neg\Box_e b$

[3] Degano, Marty, Ramotowska, MA, Breheny, Romoli, Sudo. SuB & XPRAG, 2023.

Two derivations of full ignorance

1. Neo-Gricean derivation [Sauerland 2004]

   (i) Uncertainty derived through quantity reasoning

   (18) $\alpha \vee \beta$ ASSERTION

   (19) $\{\alpha, \beta\}$ RELEVANT ALTERNATIVES

   (20) $\neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY (from QUANTITY)

   (ii) Possibility derived from uncertainty and quality about assertion

   (21) $\Box_e(\alpha \vee \beta)$ QUALITY ABOUT ASSERTION

   (22) $\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY

2. Nihil derivation

   (i) Possibility derived as neglect-zero effect

   (23) $\alpha \vee \beta$ ASSERTION

   (24) $\Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY (from NEGLECT-ZERO)

   (ii) Uncertainty derived from possibility and scalar reasoning

   (25) $\neg(\alpha \wedge \beta)$ SCALAR IMPLICATURE

   (26) $\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY

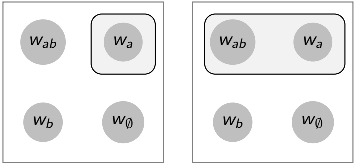
Novel hypothesis: neglect-zero

Contrasting predictions of competing accounts of ignorance

- Neo-Gricean: No possibility without uncertainty
- Neglect-zero: Possibility derived independently from uncertainty

Experimental findings [Degano *et al* 2023]

- Using adapted mystery box paradigm, compared conditions in which
  - both uncertainty and possibility are false [zero-model]
  - uncertainty false but possibility true [no-zero, no-uncertain model]
- Less acceptance when possibility is false (95% vs 34%)
- Evidence that possibility can arise without uncertainty
- A challenge for the traditional neo-gricean approach



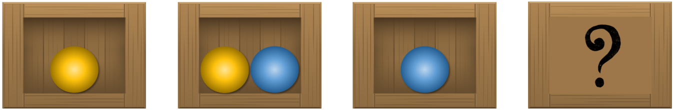
(a) zero-model (b) no-uncertain

Figure: Models for $(a \vee b)$

The mystery box paradigm[4]

The display

3 overt boxes containing colored balls and 1 mystery box.



The rule

The mystery box has the same contents as one of the overt boxes.

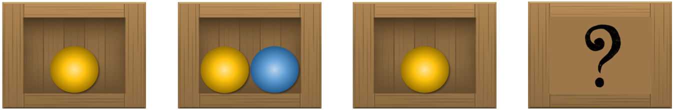
Examples

'The mystery box contains a yellow ball or a blue ball.'	$Y \vee B$	**True**
'The mystery box contains a yellow ball or a green ball.'	$Y \vee G$	**False**

[4] Adapted from Noveck 2001; see also Crnic et al. 2015

TARGET-1 (no-uncertain)

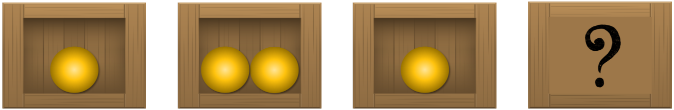
'The mystery box contains a yellow ball or a blue ball.' $Y \vee B$



UNCERTAINTY	$\neg\Box_e Y \wedge \neg\Box_e B$	**False**
POSSIBILITY	$\Diamond_e Y \wedge \Diamond_e B$	**True**

TARGET-2 (zero-model)

'The mystery box contains a yellow ball or a blue ball.' $Y \vee B$



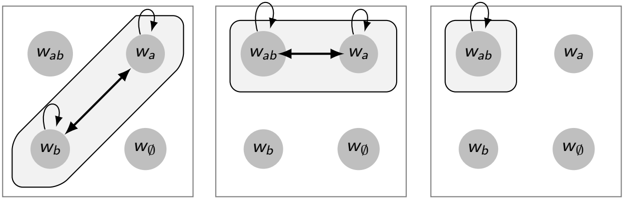
UNCERTAINTY	$\neg\Box_e Y \wedge \neg\Box_e B$	**False**
POSSIBILITY	$\Diamond_e Y \wedge \Diamond_e B$	**False**

Hypothesis
If POSSIBILITY cannot arise without UNCERTAINTY, there should be no difference in responses between TARGET-1 and TARGET-2.
Results
Target-2 far more rejected (66%) than Target-1 (5%)

Conclusions

- Evidence that possibility can arise without uncertainty
- A challenge for the traditional neo-gricean approach

Neglect-zero and no-split



(a) scalar (b) no-uncertain (c) no-split

Figure: Models for enriched $[a \vee b]^+$.

- $\{w_{ab}\}$ is a no-split verifier for the disjunction: no alternatives entertained;
- No-split conjecture: only no-split verifiers accessible to 'conjunctive' pre-school children [Klochowicz, Sbardolini, MA, 2024]
- Implementation: uses flattening operator F

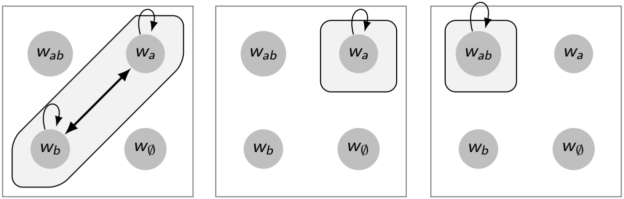
$$M, s \models \mathrm{F}\phi \text{ iff for all } w \in s : M, \{w\} \models \phi$$

Flattening $\mapsto$ formulas always interpreted wrt to singleton substates

- Combination of no-split and **no-zero** yields conjunctive *or*:

$$\mathrm{F}[\alpha \vee \beta]^+ \equiv \alpha \wedge \beta$$
$$\neg\mathrm{F}[\alpha \vee \beta]^+ \equiv \neg\alpha \wedge \neg\beta$$

Illustration: combination of no-split and no-zero yields conjunctive *or*



(a) no-zero & split (b) zero & no-split (c) no-zero & no-split

Figure: Models for $(a \vee b)$

(27) It is raining or snowing.

a. No-zero & split: [//// //// //// | ❄❄❄] [adult-like]

b. Zero & (no-)split: [//// //// ////] [logician]

c. No-zero & no-split: [//// //// //// **&** ❄❄❄] ['conjunctive' children]

Predicted inferences

- $[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$; $\not\models \alpha \wedge \beta$ [adult-like]
- $\mathrm{F}(\alpha \vee \beta) \equiv a \vee b \not\models \Diamond_e \alpha \wedge \Diamond_e \beta$; $\not\models \alpha \wedge \beta$ [logician]
- $[\mathrm{F}(\alpha \vee \beta)]^+ \models \alpha \wedge \beta$ ['conjunctive' children]

BSML^{F}: definitions

Flattening

$\mathrm{F}\phi$ is supported only if ϕ supported by all singleton subsets

$$M, s \models \mathrm{F}\phi \text{ iff for all } w \in s : M, \{w\} \models \phi$$

No-split

For F-free α, $[\alpha]^1$ defined as follows:

$$\begin{aligned}
[p]^1 &= \mathrm{F}p \\
[\neg\alpha]^1 &= \mathrm{F}\neg[\alpha]^1 \\
[\alpha \vee \beta]^1 &= \mathrm{F}([\alpha]^1 \vee [\beta]^1) \\
[\alpha \wedge \beta]^1 &= \mathrm{F}([\alpha]^1 \wedge [\beta]^1) \\
[\Diamond\alpha]^1 &= \mathrm{F}\Diamond[\alpha]^1
\end{aligned}$$

Two views on conjunctive 'or'

- Two explanations of conjunctive 'or' in pre-school children:
  - Grammatical view: conjunctive children can compute implicatures but do not have access to scalar alternatives (or $<$ and);
  - Nihil: conjunctive behaviour derives from the combination of two cognitive bias: no-zero and no-split.

	conjunctive *or*	**inclusive** *or*	**exclusive** *or*
Grammatical	exh-alt [✓]	exh-alt [no]	scalar-alt [✓]
Nihil	zero [no] & split [no]	split [✓] (or zero [✓])	split [✓] & scalar reasoning [✓]

- Two different acquisition patterns:
  - Grammatical view:
    inclusive *or* $<$ conjunctive *or* $<$ exclusive *or*
  - Nihil:
    conjunctive *or* $<$ inclusive *or* $<$ exclusive *or*
- New experiments needed to decide between different views

Free Choice

The paradox of free choice

- Free choice permission in natural language:

(28) You may (A or B) $\rightsquigarrow$ You may A

- But (29) not valid in standard deontic logic [von Wright 1968]:

(29) $\Diamond(\alpha \vee \beta) \rightarrow \Diamond\alpha$ [Free Choice (FC) Principle]

- Plainly making FC Principle valid, for example by adding it as an axiom, would not do [Kamp 1973]:

(30)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by classical reasoning]
3. $\Diamond b$ [from 2, by FC principle]

- The step leading to 2 in (30) uses the following valid principle:

(31) $\Diamond\alpha \rightarrow \Diamond(\alpha \vee \beta)$

- Natural language counterpart of (31), however, seems invalid:

(32) You may post this letter $\not\rightsquigarrow$ You may post this letter or burn it. [Ross's paradox]

$\Rightarrow$ Intuitions on natural language in direct opposition to the principles of classical logic

Reactions to paradox

- Paradox of FC Permission:

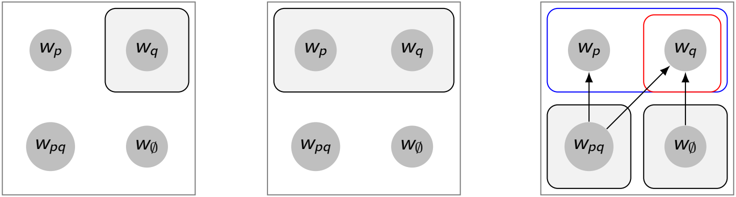
(33)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by addition + monotonicity]
3. $\Diamond b$ [from 2, by FC principle]

- Pragmatic (neo-Gricean) solutions [$\Rightarrow$ keep logic as is]
  - FC inferences are pragmatic inferences (conversational implicatures)
  - $\Rightarrow$ step leading to 3 is unjustified
- Grammatical solutions [$\Rightarrow$ keep logic as is]
  - FC inferences (like scalar implicatures) result from the application of covert grammatical operators
  - $\Rightarrow$ step leading to 3 is unjustified
- Semantic solutions [$\Rightarrow$ change the logic]
  - FC inferences are semantic entailments (e.g., MA 2007)
  - $\Rightarrow$ step leading to 3 is justified, but step leading to 2 is no longer valid (or transitivity fails)
- Present proposal [$\Rightarrow$ change the logic]
  - FC inferences as **neglect-zero** effects only derived for enriched $[\phi]^+$

(34)

$$\begin{array}{lccc} 1. & \Diamond\alpha & & \\ & \Downarrow & & \\ 2. & \Diamond(\alpha \vee \beta) & \neq & [\Diamond(\alpha \vee \beta)]^+ \\ & & & \Downarrow \\ 3. & & & \Diamond\beta \end{array}$$

Illustration: possibility & narrow scope free choice



(a) $\{w_q\} \not\models [p \vee q]^+$

(b) $\{w_p, w_q\} \models [p \vee q]^+$

(c) $\{w_\emptyset\} \not\models [\Diamond(p \vee q)]^+$; $\{w_{pq}\} \models [\Diamond(p \vee q)]^+$

Relevant semantic clauses

$$
\begin{array}{rll}
M, s \models p & \text{iff} & \text{for all } w \in s : V(w,p) = 1 \\
M, s \models \phi \vee \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \ \&\ M, t \models \phi \ \&\ M, t' \models \psi \\
M, s \models \Diamond\phi & \text{iff} & \text{for all } w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \&\ M, t \models \phi \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset
\end{array}
$$

Neglect-zero enrichment

$$
\begin{array}{rcl}
[\Diamond(p \vee q)]^+ & := & \Diamond[p \vee q]^+ \wedge \mathrm{NE} \\
[p \vee q]^+ & := & ((p \wedge \mathrm{NE}) \vee (q \wedge \mathrm{NE})) \wedge \mathrm{NE}
\end{array}
$$

Novel hypothesis: neglect-zero

Comparison with competing accounts of FC inference

	NS FC	Dual Prohib	Universal FC	Double Neg	WS FC
Neo-Gricean	yes	yes	no	?	no
Grammatical	yes	yes*	yes	no*	no*
Semantic	yes	no*	yes	no*	no
Neglect-zero	yes	yes	yes	yes	yes

Argument in favor of neglect-zero hypothesis

- Empirical coverage: FC sentences give rise to a complex pattern of inferences

(35) a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]
b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]
c. $\forall x \Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]
d. $\neg\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Double Negation FC]
e. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

- Captured by neglect-zero approach implemented in BSML[5]
- Most other approaches need additional assumptions

[5] MA (2022). Logic and conversation: the case of FC. *Sem & Pra*, 15(5).

The data

(36) Dual Prohibition [Alonso-Ovalle 2006, Marty *et al.* 2021]

a. You are not allowed to eat the cake or the ice-cream.
$\rightsquigarrow$ You are not allowed to eat either one.

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

(37) Universal FC [Chemla 2009]

a. All of the boys may go to the beach or to the cinema.
$\rightsquigarrow$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x \Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

(38) Double Negation FC [Gotzner *et al.* 2020]

a. Exactly one girl cannot take Spanish or Calculus.
$\rightsquigarrow$ One girl can take neither of the two and each of the others can choose between them.

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

(39) Wide Scope FC [Zimmermann 2000, Hoeks *et al.* 2017]

a. Detectives may go by bus or they may go by boat.
$\rightsquigarrow$ Detectives may go by bus and may go by boat.

b. Mr. X might be in Victoria or he might be in Brixton.
$\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.

c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

BSML results: negation

- **Negation** facts follow from adopted bilateralism (we validate De Morgan laws and $\neg\neg$-elimination):
  - Adding NE vacuous under single negation: $\mapsto$ Dual Prohibition

$$\neg(\alpha \wedge \text{NE}) \equiv \neg\alpha \vee \neg\text{NE} \equiv \neg\alpha \vee \bot \equiv \neg\alpha$$

  - Adding NE non-vacuous under double negation: $\mapsto$ Double Negation FC

$$\neg\neg(\alpha \wedge \text{NE}) \equiv \alpha \wedge \text{NE} \not\equiv \neg\neg\alpha$$

$\Rightarrow$ Failure of replacement under negation (in fact analogue of Burgess' negation theorem holds, Anttila22):

$$\phi \equiv \psi \not\Rightarrow \neg\phi \equiv \neg\psi$$

$\Rightarrow$ Empirically correct predictions:

- FC effects systematically disappear under **single negation** $\mapsto$ Dual Prohibition [Alonso-Ovalle 2006]
- But speakers draw FC inferences under **double negation** $\mapsto$ Double Negation FC [Romoli & Santorio 2019, Gotzner et al. 2020]

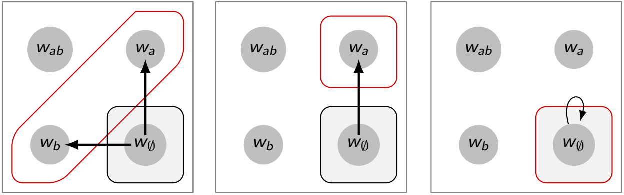
$\Rightarrow$ Without replacement failure impossible to capture these data

BSML: free choice facts

- **Free choice** results rely on relational notion of modality:
  - A state s supports $\Diamond\phi$ iff for every w in s there is a non-empty subset of the set of worlds accessible from w which supports ϕ:

$$M, s \models \Diamond\phi \text{ iff } \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \& \ M, t \models \phi$$

- **Narrow and wide scope** FC
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide-scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ [if R is indisputable]



(d) No-zero verifier (e) Zero-model (f) Falsifier

Figure: Models for $\Diamond(a \vee b)$ and $\Diamond a \vee \Diamond b$ (R is indisputable).

- Zero-model $\{w_\emptyset \rightarrow w_a\}$ supports $\Diamond a \vee \Diamond b$, but does not support $[\Diamond a \vee \Diamond b]^+$

BSML: free choice facts

- **Free choice** results rely on relational notion of modality:
  - A state s supports $\Diamond\phi$ iff for every w in s there is a non-empty subset of the set of worlds accessible from w which supports ϕ:

$$M, s \models \Diamond\phi \text{ iff } \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \& \ M, t \models \phi$$

- **Failure of wide scope** FC (R is not indisputable).

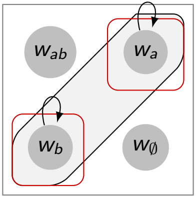


Figure: $s \models [\Diamond a \vee \Diamond b]^+$, but $s \not\models \Diamond a$ (and also $s \not\models [\Diamond(a \vee b)]^+$)

Neglect-zero effects in BSML: predictions

After enrichments

- We derive both wide and narrow scope FC inferences for pragmatically enriched formulas:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before enrichments

- The NE-free fragment of BSML is equivalent to classical modal logic:

$$\alpha \models_{BSML} \beta \text{ iff } \alpha \models_{CML} \beta \qquad [\text{if } \alpha, \beta \text{ are NE-free}]$$

- But we can capture infelicity of epistemic contradictions (Yalcin, 2007) by putting team-based constraints on accessibility relation:
  1. Epistemic contradiction: $\Diamond\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

BSML predictions: epistemic and deontic FC

- Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
- Wide-scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ [if *R* is indisputable]

Epistemic modals

- *R* is **state-based**, therefore always indisputable:

(40) He might be in either London or Paris. [+fc, narrow]
(41) He might be in London or he might be in Paris. [+fc, wide]

$\Rightarrow$ narrow and wide scope FC always predicted for pragmatically enriched epistemics

Deontic modals

- *R* **sometimes indisputable**, e.g. in performative uses

$\Rightarrow$ narrow scope FC always predicted for enriched deontics

$\Rightarrow$ wide scope FC only if speaker is informed about what is permitted/obligatory

[Cremers et al 2017]

Further consequence: all cases of (overt) FC cancellations involve a wide scope configuration in a context where indisputability is not warranted

BSML predictions: overt FC cancellations

- Examples of overt FC cancellations:

(42) You may eat either the cake or the ice-cream, I don't know which $\not\rightsquigarrow$ You may eat the cake

(43) You may eat either the cake or the ice-cream, it depends on what John has taken $\not\rightsquigarrow$ You may eat the cake [Kaufmann 2016]

- Sluicing in (42) and inquisitive *it* in (43) arguably triggers wide scope disjunction in their antecedent [Fusco 2019, Pinton & MA 2022]

(44) You may eat either the cake or the ice-cream, I don't know which (you may eat). [wide, –fc]

(45) You may eat either the cake or the ice-cream, it (= what you may eat) depends on what John has taken. [wide, –fc]

- Sketch of analysis (in BSML^{F} + inquisitive disjunction):

(a) which/what you may eat $\mapsto \Diamond\alpha \vee\!\!\!\vee \Diamond\beta \equiv \boxed{\begin{array}{c}\Diamond\alpha \\ \hline \Diamond\beta\end{array}}$

(b) $\Diamond\alpha \vee \Diamond\beta \equiv \mathrm{F}\ \boxed{\begin{array}{c}\Diamond\alpha \\ \hline \Diamond\beta\end{array}} \not\equiv \Diamond(\alpha \vee \beta)$

Neglect-zero effects in BSML: further predictions

- Modal **D-inferences** are derived: [Ramotowska et al 2022]
  - $[\Box(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
- But **negative** FC is not predicted: [Marty et al 2022]
  - $[\neg\Box(\alpha \wedge \beta)]^+ \not\models \Diamond\neg\alpha \wedge \Diamond\neg\beta$

- In BSML logically equivalent sentences can have different neglect-zero effects, i.e., these effects are detachable:

$$\Diamond(\neg\alpha \vee \neg\beta) \quad \equiv \quad \neg\Box(\alpha \wedge \beta)$$

But:

$$[\Diamond(\neg\alpha \vee \neg\beta)]^+ \quad \models \quad \Diamond\neg\alpha \wedge \Diamond\neg\beta$$
$$[\neg\Box(\alpha \wedge \beta)]^+ \quad \not\models \quad \Diamond\neg\alpha \wedge \Diamond\neg\beta$$

Negative FC (Marty et al., 2021, 2022)

- **Experimental research**: negative FC inferences exist but appear to be less available than positive FC:

(46) Negative FC

a. It is not required that Mia buys both apples and bananas $\rightsquigarrow$ It is not required that Mia buys apples and that Mia buys bananas

b. $\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$ $(\equiv \Diamond\neg\alpha \wedge \Diamond\neg\beta)$

- BSML^+: BSML + global neglect-zero enrichment

$$\alpha \models_{BSML^+} \beta \text{ iff } [\alpha]^+ \models_{BSML} [\beta]^+$$

- Perfect match between BSML^+ and experimental findings:

			BSML^+
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**strong**	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \Diamond\neg\alpha \wedge \Diamond\neg\beta$	**weak**	-
D-inference	$\Box(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**strong**	+
Negative DI	$\neg\Diamond(\alpha \wedge \beta) \rightsquigarrow \Diamond\neg\alpha \wedge \Diamond\neg\beta$	**weak**	-
Low Negative FC	$\Diamond(\neg\alpha \vee \neg\beta) \rightsquigarrow \Diamond\neg\alpha \wedge \Diamond\neg\beta$	**strong**	+

Table: Comparison BSML^+ and experimental findings.

Comparison with two recent approaches

- Goldstein 2019: FC inferences derived by adding a homogeneity presupposition to the meaning of
  - possibility modal [alternative-based account, Gold19A]
  - disjunction [dynamic account, Gold19B]
- Bar-Lev & Fox 2020: FC inference derived by application of an exhaustivity operator (which includes alternatives on top of negating all the innocently excludable ones) [BLF20]

		BSML^+	Gold19A	Gold19B	BLF20
Positive FC	**strong**	+	+	+	+
Negative FC	**weak**	-	-	-	+
Modal Disjunction	**strong**	+	-	+	?
Negative Conjunction	**weak**	-	-	-	?
Wide Scope FC	?	+	-	+	-

Table: Comparison BSML^+ and alternative approaches

- BSML^+ & Gold19B seem the best options for strong inferences but needs to be supplemented with a theory deriving weak inferences;
- NEXT: Within BSML we can derive both weak and strong inference patterns: $\text{BSML}^+ \mapsto$ strong & $\text{BSML}^* \mapsto$ weak

Modelling neglect-zero effects: different implementations

- More ways to model neglect-zero effects:
  - Syntactically, via pragmatic enrichment function $[\ ]^+$ defined in terms of NE
    $\mapsto$ BSML^+
  - Model-theoretically, by ruling out $\emptyset$ from the set of possible states
    $\mapsto$ BSML^*
- Both implementations derive:
  - $\mapsto$ FC effects (narrow and wide scope FC, the latter with restrictions);
  - $\mapsto$ cancellations of FC effects under negation (dual prohibition);
  - $\mapsto$ possibility inferences (weak ignorance);
  - $\mapsto$ conjunctive *or* in combination with flattening F.
- But empirical and conceptual differences:
  1. Only BSML^* predicts
     - Negative FC: $\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$ [Marty et al]
     - Homogeneity effects with F: $\neg(\alpha \wedge \beta) \rightsquigarrow \neg\alpha \wedge \neg\beta$ [Sbardolini23]
  2. Only in BSML^+, where $\emptyset$ is part of the building blocks, locality and suspension of neglect-zero effects can be modeled
- Conjecture: neglect-zero can cause two kinds of effects:
  - (i) weak non-detachable effects (modelled by BSML^*);
  - (ii) more robust detachable effects (modelled by *BSML*$^+$).

Non-detachable neglect-zero effects: BSML^*

- BSML^*: like BSML, but $\emptyset$ is not among the possible states;
- **Fact:** Let α, β be classical $\neg$-free formulas. Then

$$\alpha \models_{BSML^*} \beta \text{ iff } \alpha \models_{BSML^+} \beta$$

- But this does not hold in general. In BSML^*, FC inferences generated also for negative conjunctions ($\Rightarrow$ Negative FC):

$$\neg\Box(\alpha \wedge \beta) \models_{BSML^*} \neg\Box\alpha \wedge \neg\Box\beta$$
$$\neg(\alpha \wedge \beta) \models_{BSML^*} \Diamond_e\neg\alpha \wedge \Diamond_e\neg\beta$$

- Conjecture: BSML^* characterises purely pragmatic neglect-zero effects deriving from the global omission of the empty set (**low cost pragmatics**)

			BSML^+	BSML^*
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	**s**	+	+
Dual Prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	**s**	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	**w**	-	+
Modal Disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond_e\alpha \wedge \Diamond_e\beta$	**s**	+	+
Negative Conjunction	$\neg(\alpha \wedge \beta) \rightsquigarrow \Diamond\neg\alpha \wedge \Diamond\neg\beta$	**w**	-	+

Global suspension of neglect-zero effects: $\text{BSML}^{\emptyset}$

- Despite their cognitive cost, zero-models are not always neglected.
- In logico-mathematical reasonings, neglect-zero effects are globally suspended:

(47) A. Therefore, A or B.
(48) If A then B. Therefore, if not B then not A.

- Global suspension modeled by $\text{BSML}^{\emptyset}$ = NE-free fragment of BSML

		$\text{BSML}^{\emptyset}$	BSML^{*}
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	-	+
Addition	$\alpha \models \alpha \vee \beta$	+	-
Contraposition	$\alpha \models \beta \;\Rightarrow\; \neg\beta \models \neg\alpha$	+	-

- In $\text{BSML}^{\emptyset}$ (= classical logic), $\emptyset$ plays an essential role:

  The point about zero is that we do not need to use it in the operations of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all the cardinals, and its use is only forced on us by the needs of cultivated modes of thought. (Alfred North Whitehead, quoted by Nieder 2016)

- One more conjecture: at least in part, divergence between everyday and logico-mathematical reasoning might be due to a neglect-zero tendency.

The resulting picture

- A pluralism of systems which can be used to model interpretation strategies & reasoning styles people may adopt in different circumstances:
  1. BSML$^{\emptyset}$: modelling logical-mathematical reasoning where neglect-zero effects are obviated;
  2. BSML^{+}: modelling strong (detachable) neglect-zero effects;
  3. BSML*: modelling weak (global, non-detachable) neglect-zero effects;
  4. BSML^{*F}: modelling conjunctive children
  5. ...
- Experimentally testable predictions arising from these conjectures

			BSML$^{\emptyset}$	BSML^{+}	BSML*
NS FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	s	-	+	+
Dual prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	s	+	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	w	-	-	+
WS FC	$\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	?	-	+	+

Table: Comparison BSML$^{\emptyset}$, BSML^{+} and BSML*.

Conclusions

- Free choice and ignorance: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal: stronger meanings consequences of cognitive biases
  - FC and ignorance as neglect-zero effects

    Literal meanings (NE-free fragment) + cognitive factor (NE) $\Rightarrow$ FC & possibility inferences

  - Conjunctive *or* as no-split effect

    Literal meanings (NE-free fragment) + cognitive factors (NE, F) $\Rightarrow$ conjunctive *or*

- Implementation in variants of BSML (a team-based modal logic)
  - BSML* vs BSML$^+$ vs BSML$^\emptyset$

Collaborators & related (future) research

Logic

Proof theory (Anttila, Yang); expressive completeness (Anttila, Yang, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); BiUS & qBiUS (MA); typed BSML (Muskens); connexive logic (Knudstorp, MA);. . .

Language

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs); questions (Klochowicz); quantifiers (Ramotowska, Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo); . . .

THANK YOU![6]

[6] This work was supported by NWO OC project *Nothing is Logical* (NihiL) (grant no 406.21.CTW.023).

Exercises

1. Check whether the following statements are correct:
    1. $p \wedge \neg p \equiv \text{NE} \wedge \neg \text{NE}$
    2. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
    3. $[\neg(p \wedge q)]^+ \equiv [\neg p \vee \neg q]^+$
    4. $[\text{F}\neg(a \wedge b)]^+ \equiv [\text{F}(\neg a \wedge \neg b)]^+$
    5. $[\text{F}\neg(a \wedge b)]^* \equiv [\text{F}(\neg a \wedge \neg b)]^*$
2. BSML violates the principle of replacement under negation. Give examples of ϕ and ψ such that $\phi \equiv \psi$ but $\neg\phi \not\equiv \neg\psi$