Philosophical and linguistic applications of team semantics

Class 3: (q)BSML

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Outline of course

- Day 1. Team semantics
- Day 2. BSML: ignorance and free choice inferences
- Day 3. **(q)BSML & guest lecture dr. Yan** ⇐
- Day 4. Inquisitive semantics: questions and inquisitive attitude verbs
- Day 5. Dependence logic: marked indefinites cross-linguistically

Plan of today

- Guest lecture dr. Yan
- Variants of BSML: BSML$^+$ vs BSML* vs BSML$^\emptyset$ vs BSML$^*_F$
- Preliminary results FC questions experiment
- Questions on BSML
- Solution exercises day one
- qBSML: modified numerals

Literature on qBSML

- Aloni, M and Ormondt, P van. 2023. "Modified numerals and split disjunction: the first-order case." *Journal of Logic, Language and Information* 32, 539–567.
- ...

Neglect-zero effects in BSML: predictions

After enrichments

- We derive both wide and narrow scope FC inferences for pragmatically enriched formulas:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
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- But we can capture infelicity of epistemic contradictions (Yalcin, 2007) by putting team-based constraints on accessibility relation:
  1. Epistemic contradiction: $\Diamond_e\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

Neglect-zero effects in BSML: further predictions

- Modal **D-inferences** are derived: [Ramotowska et al 2022]
  - $[\Box(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
- But **negative** FC is not predicted: [Marty et al 2022]
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  - $[\neg\Box(\alpha \wedge \beta)]^+ \not\models \Diamond\neg\alpha \wedge \Diamond\neg\beta$
- In BSML logically equivalent sentences can have different neglect-zero effects, i.e., these effects are detachable:

$$\Diamond(\neg\alpha \vee \neg\beta) \quad \equiv \quad \neg\Box(\alpha \wedge \beta)$$

But:

$$[\Diamond(\neg\alpha \vee \neg\beta)]^+ \quad \models \quad \Diamond\neg\alpha \wedge \Diamond\neg\beta$$
$$[\neg\Box(\alpha \wedge \beta)]^+ \quad \not\models \quad \Diamond\neg\alpha \wedge \Diamond\neg\beta$$

Negative FC (Marty et al., 2021, 2022)

- **Experimental research**: negative FC inferences exist but appear to be less available than positive FC:

(1) Negative FC

a. It is not required that Mia buys both apples and bananas $\rightsquigarrow$ It is not required that Mia buys apples and that Mia buys bananas

b. $\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$ $(\equiv \Diamond\neg\alpha \wedge \Diamond\neg\beta)$

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- NEXT: Using variants of BSML we can derive both weak and strong inference patterns: BSML$^+$ $\mapsto$ strong & BSML* $\mapsto$ weak

Modelling neglect-zero effects: different implementations

- More ways to model neglect-zero effects:
  - Syntactically, via pragmatic enrichment function $[\ ]^+$ defined in terms of $\textsc{ne}$
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- Both implementations derive:
  - $\mapsto$ FC effects (narrow and wide scope FC, the latter with restrictions);
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Dual Prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	**s**	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	**w**	-	+
Modal Disjunction	$\alpha \vee \beta \rightsquigarrow \Diamond_e\alpha \wedge \Diamond_e\beta$	**s**	+	+
Negative Conjunction	$\neg(\alpha \wedge \beta) \rightsquigarrow \Diamond\neg\alpha \wedge \Diamond\neg\beta$	**w**	-	+

Global suspension of neglect-zero effects: BSML$^{\emptyset}$

- Despite their cognitive cost, zero-models are not always neglected.
- In logico-mathematical reasonings, neglect-zero effects are globally suspended:

(2) A. Therefore, A or B.

(3) If A then B. Therefore, if not B then not A.

- Global suspension modeled by BSML$^{\emptyset}$ (NE-free fragment of BSML)

		BSML$^{\emptyset}$	BSML*
Positive FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	-	+
Addition	$\alpha \models \alpha \vee \beta$	+	-
Contraposition	$\alpha \models \beta \;\Rightarrow\; \neg\beta \models \neg\alpha$	+	-

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- In BSML$^{\emptyset}$ (= classical logic), $\emptyset$ plays an essential role:

The point about zero is that we do not need to use it in the operations of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all the cardinals, and its use is only forced on us by the needs of cultivated modes of thought. (Alfred North Whitehead, quoted by Nieder 2016)

- One more conjecture: at least in part, divergence between everyday and logico-mathematical reasoning might be due to a neglect-zero tendency.

The resulting picture

- A pluralism of systems which can be used to model interpretation strategies & reasoning styles people may adopt in different circumstances:
  1. BSML$^{\emptyset}$: modelling logical-mathematical reasoning where neglect-zero effects are obviated;
  2. BSML^{+}: modelling strong (detachable) neglect-zero effects;
  3. BSML*: modelling weak (global, non-detachable) neglect-zero effects;
  4. BSML$^{*}_{F}$: modelling pre-school 'conjunctive' children's reasoning style (deriving conjunctive strengthening of disjunctions)
  5. ...
- Experimentally testable predictions arising from these conjectures

			BSML$^{\emptyset}$	BSML^{+}	BSML*
NS FC	$\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	s	-	+	+
Dual prohibition	$\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$	s	+	+	+
Negative FC	$\neg\Box(\alpha \wedge \beta) \rightsquigarrow \neg\Box\alpha \wedge \neg\Box\beta$	w	-	-	+
WS FC	$\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$	?	-	+	+

Table: Comparison BSML$^{\emptyset}$, BSML^{+} and BSML*.

Conclusions

- Free choice and ignorance: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal: stronger meanings consequences of cognitive biases
  - FC and ignorance as neglect-zero effects

    Literal meanings (NE-free fragment) + cognitive factor (NE) $\Rightarrow$ FC & possibility inferences

  - Conjunctive *or* as no-split effect

    Literal meanings (NE-free fragment) + cognitive factors (NE, F) $\Rightarrow$ conjunctive *or*

- Implementation in BSML (a team-based modal logic)
- Different reasoning styles modelled by different variants of BSML:
  - BSML$^+$ vs BSML* vs BSML$^\emptyset$ vs BSML$^*_F$

Preliminary results Mandarin FC question experiment

(4) May I do A or B?

a. Yes $\rightsquigarrow$ Both A and B allowed (Free Choice)

b. No $\rightsquigarrow$ Neither A nor B allowed (Dual Prohibition)

- Dual Prohibition: 80% (93% in the English data)
- Free Choice: 61% (75% in the English data)
- The zero models (one allowed contexts) take longer (9 sec) than the others (4.5 sec)
- The conversations containing negation take longer (6.4 sec) than ones with 'yes' (5.5 sec)

Dependence Logic: FO + $dep(\vec{x}, \vec{y})$

Language

[Väänänen 2007]

$$\alpha ::= x = x \mid R(\vec{x}) \mid \neg\alpha \mid \alpha \vee \alpha \mid \alpha \wedge \alpha \mid \exists x\alpha \mid \mid \forall x\alpha \quad \text{(with } x \in Var\text{)}$$

$$\phi ::= \alpha \mid \neg\alpha \mid \phi \vee \phi \mid \phi \wedge \phi \mid \exists x\phi \mid \forall x\phi \mid dep(\vec{x}, \vec{x})$$

[NB: $\neg dep(x, y))$ is not a well-formed formula]

Team Semantics

Let $M = \{D, I\}$ be a FO model and T be a team, i.e. a set of assignments $i : V \rightarrow D$, with $V \subseteq Var$

$M \models_T \alpha$	$\Leftrightarrow$	for all $j \in T$: $M \models_j \alpha$, when α is a FO formula
$M \models_T \neg\alpha$	$\Leftrightarrow$	for all $j \in T$: $M \not\models_j \alpha$, whenever α is a FO formula
$M \models_T \phi \wedge \psi$	$\Leftrightarrow$	$M \models_T \phi$ and $M \models_T \psi$
$M \models_T \phi \vee \psi$	$\Leftrightarrow$	there are S, S' with $T = S \cup S'$ s.t. $M \models_S \phi$ & $M \models_{S'} \psi$
$M \models_T \forall x\phi$	$\Leftrightarrow$	$M \models_{T[x]} \phi$, where $T[x] = \{i[d/x] : i \in T \text{ and } d \in D_M\}$
$M \models_T \exists x\phi$	$\Leftrightarrow$	there is a function $f : T \rightarrow \wp(D_M) \setminus \emptyset$, s.t. $M \models_{T[f/x]} \phi$, where $T[f/x] = \{i[d/x] : i \in T \text{ and } d \in f(i)\}$
$M \models_T dep(\vec{x}, \vec{y})$	$\Leftrightarrow$	for all $i, j \in T$: $i(\vec{x}) = j(\vec{x}) \Rightarrow i(\vec{y}) = j(\vec{y})$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

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1.3 $M \models_T dep(v, x)$

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i_3	b	a	b	d
i_4	b	a	d	d

1.1 $M \models_T dep(x, y)$ (yes)

1.2 $M \models_T dep(x, v)$ (yes)

1.3 $M \models_T dep(v, x)$ (no)

1.4 $M \models_T dep(xy, z)$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

1.1 $M \models_T dep(x, y)$ (yes)
1.2 $M \models_T dep(x, v)$ (yes)
1.3 $M \models_T dep(v, x)$ (no)
1.4 $M \models_T dep(xy, z)$ (no)
1.5 $M \models_T dep(\emptyset, v)$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

   1.1 $M \models_T dep(x, y)$ (yes)
   1.2 $M \models_T dep(x, v)$ (yes)
   1.3 $M \models_T dep(v, x)$ (no)
   1.4 $M \models_T dep(xy, z)$ (no)
   1.5 $M \models_T dep(\emptyset, v)$ (yes)

2. Determine whether the following claims are correct given T as depicted.

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

   2.1 $M \models_T x \leq z$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

   1.1 $M \models_T dep(x, y)$ (yes)
   1.2 $M \models_T dep(x, v)$ (yes)
   1.3 $M \models_T dep(v, x)$ (no)
   1.4 $M \models_T dep(xy, z)$ (no)
   1.5 $M \models_T dep(\emptyset, v)$ (yes)

2. Determine whether the following claims are correct given T as depicted.

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

   2.1 $M \models_T x \leq z$ (no)
   2.2 $M \models_T x \leq z \vee z \leq y$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

   1.1 $M \models_T dep(x, y)$ (yes)
   1.2 $M \models_T dep(x, v)$ (yes)
   1.3 $M \models_T dep(v, x)$ (no)
   1.4 $M \models_T dep(xy, z)$ (no)
   1.5 $M \models_T dep(\emptyset, v)$ (yes)

2. Determine whether the following claims are correct given T as depicted.

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

   2.1 $M \models_T x \leq z$ (no)
   2.2 $M \models_T x \leq z \vee z \leq y$ (yes)
   2.3 $M \models_T \exists v\ v \leq z$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

1.1 $M \models_T dep(x, y)$ (yes)
1.2 $M \models_T dep(x, v)$ (yes)
1.3 $M \models_T dep(v, x)$ (no)
1.4 $M \models_T dep(xy, z)$ (no)
1.5 $M \models_T dep(\emptyset, v)$ (yes)

2. Determine whether the following claims are correct given T as depicted.

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

2.1 $M \models_T x \leq z$ (no)
2.2 $M \models_T x \leq z \vee z \leq y$ (yes)
2.3 $M \models_T \exists v\ v \leq z$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

   1.1 $M \models_T dep(x, y)$ (yes)
   1.2 $M \models_T dep(x, v)$ (yes)
   1.3 $M \models_T dep(v, x)$ (no)
   1.4 $M \models_T dep(xy, z)$ (no)
   1.5 $M \models_T dep(\emptyset, v)$ (yes)

2. Determine whether the following claims are correct given T as depicted.

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

   2.1 $M \models_T x \leq z$ (no)
   2.2 $M \models_T x \leq z \vee z \leq y$ (yes)
   2.3 $M \models_T \exists v \; v \leq z$ (yes)
   2.4 $M \models_T \exists v(v \leq z \wedge y \leq v)$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

   1.1 $M \models_T dep(x, y)$ (yes)
   1.2 $M \models_T dep(x, v)$ (yes)
   1.3 $M \models_T dep(v, x)$ (no)
   1.4 $M \models_T dep(xy, z)$ (no)
   1.5 $M \models_T dep(\emptyset, v)$ (yes)

2. Determine whether the following claims are correct given T as depicted.

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

   2.1 $M \models_T x \leq z$ (no)
   2.2 $M \models_T x \leq z \vee z \leq y$ (yes)
   2.3 $M \models_T \exists v\ v \leq z$ (yes)
   2.4 $M \models_T \exists v (v \leq z \wedge y \leq v)$ (no)
   2.5 $M \models_T \forall v\ z \leq v$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

   1.1 $M \models_T dep(x, y)$ (yes)
   1.2 $M \models_T dep(x, v)$ (yes)
   1.3 $M \models_T dep(v, x)$ (no)
   1.4 $M \models_T dep(xy, z)$ (no)
   1.5 $M \models_T dep(\emptyset, v)$ (yes)

2. Determine whether the following claims are correct given T as depicted.

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

   2.1 $M \models_T x \leq z$ (no)
   2.2 $M \models_T x \leq z \vee z \leq y$ (yes)
   2.3 $M \models_T \exists v\ v \leq z$ (yes)
   2.4 $M \models_T \exists v (v \leq z \wedge y \leq v)$ (no)
   2.5 $M \models_T \forall v\ z \leq v$ (no)

3. Show that $dep(x, y) \vee dep(x, y) \not\models dep(x, y)$

Exercises (Dependence logic)

1. Determine whether the following claims are correct given T as depicted.

T	x	y	z	v
i_1	a	c	a	d
i_2	a	c	a	d
i_3	b	a	b	d
i_4	b	a	d	d

1.1 $M \models_T dep(x, y)$ (yes)
1.2 $M \models_T dep(x, v)$ (yes)
1.3 $M \models_T dep(v, x)$ (no)
1.4 $M \models_T dep(xy, z)$ (no)
1.5 $M \models_T dep(\emptyset, v)$ (yes)

2. Determine whether the following claims are correct given T as depicted.

T	x	y	z
i_1	3	4	5
i_2	2	3	3
i_3	1	2	0
i_4	0	1	0

2.1 $M \models_T x \leq z$ (no)
2.2 $M \models_T x \leq z \vee z \leq y$ (yes)
2.3 $M \models_T \exists v\ v \leq z$ (yes)
2.4 $M \models_T \exists v(v \leq z \wedge y \leq v)$ (no)
2.5 $M \models_T \forall v\ z \leq v$ (no)

3. Show that $dep(x, y) \vee dep(x, y) \not\models dep(x, y)$

T	x	y
i_1	0	0
i_2	0	1

Inquisitive Logic: CPL + $\vee\!\!\!/$

Language

[Ciardelli & Roelofsen 2011]

$$\phi := p \mid \bot \mid \phi \to \phi \mid \phi \wedge \phi \mid \phi \vee\!\!\!/ \ \phi \qquad \text{with } p \in PROP$$

Defined connectives: $\neg\phi := \phi \to \bot$ & $\phi \vee_* \psi := \neg(\neg\phi \wedge \neg\psi)$

Team/state-based semantics

Given a model $M = (W, V)$ and an information state $s \subseteq W$,

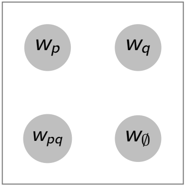
- $M, s \models p \Leftrightarrow$ for all $w \in s : V(p, w) = 1$
- $M, s \models \bot \Leftrightarrow s = \emptyset$
- $M, s \models \phi \wedge \psi \Leftrightarrow M, s \models \phi$ and $M, s \models \psi$
- $M, s \models \phi \to \psi \Leftrightarrow$ for all $t \subseteq s : t \models \phi$ implies $t \models \psi$
- $M, s \models \phi \vee\!\!\!/ \ \psi \Leftrightarrow M, s \models \phi$ or $M, s \models \psi$

Defined connectives

- $M, s \models \neg\phi \Leftrightarrow$ for all $t \subseteq s : t \not\models \phi$, unless $t = \emptyset$
- $M, s \models \phi \vee_* \psi \Leftrightarrow$ for all $w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$

Exercises (Inquisitive logic)

1. Consider the following intensional model:

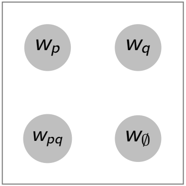


Determine whether the following statements are correct:

1.1 $\{w_p, w_{pq}\} \models p$

Exercises (Inquisitive logic)

1. Consider the following intensional model:



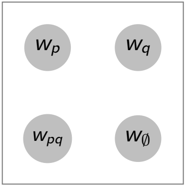
Determine whether the following statements are correct:

1.1 $\{w_p, w_{pq}\} \models p$ (yes)

1.2 $\{w_p, w_{pq}\} \models q$

Exercises (Inquisitive logic)

1. Consider the following intensional model:



Determine whether the following statements are correct:

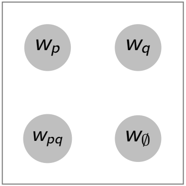
1.1 $\{w_p, w_{pq}\} \models p$ (yes)

1.2 $\{w_p, w_{pq}\} \models q$ (no)

1.3 $\{w_p, w_{pq}\} \models \neg q$

Exercises (Inquisitive logic)

1. Consider the following intensional model:



Determine whether the following statements are correct:

1.1 $\{w_p, w_{pq}\} \models p$ (yes)

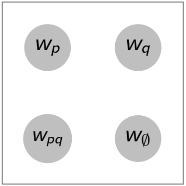
1.2 $\{w_p, w_{pq}\} \models q$ (no)

1.3 $\{w_p, w_{pq}\} \models \neg q$ (no)

1.4 $\{w_p\} \models p \vee\!\!\!\vee\, q$

Exercises (Inquisitive logic)

1. Consider the following intensional model:



Determine whether the following statements are correct:

1.1 $\{w_p, w_{pq}\} \models p$ (yes)

1.2 $\{w_p, w_{pq}\} \models q$ (no)

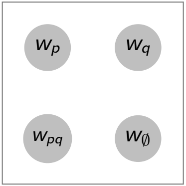
1.3 $\{w_p, w_{pq}\} \models \neg q$ (no)

1.4 $\{w_p\} \models p \mathbin{\vee\!\!\!\!/} q$ (yes)

1.5 $\{w_p, w_q\} \models p \mathbin{\vee\!\!\!\!/} q$

Exercises (Inquisitive logic)

1. Consider the following intensional model:



Determine whether the following statements are correct:

1.1 $\{w_p, w_{pq}\} \models p$ (yes)

1.2 $\{w_p, w_{pq}\} \models q$ (no)

1.3 $\{w_p, w_{pq}\} \models \neg q$ (no)

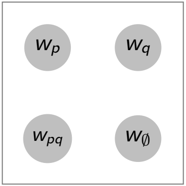
1.4 $\{w_p\} \models p \vee\!\!\!\vee\, q$ (yes)

1.5 $\{w_p, w_q\} \models p \vee\!\!\!\vee\, q$ (no)

1.6 $\{w_p, w_{pq}\} \models p \rightarrow q$

Exercises (Inquisitive logic)

1. Consider the following intensional model:



   Determine whether the following statements are correct:

   1.1 $\{w_p, w_{pq}\} \models p$ (yes)

   1.2 $\{w_p, w_{pq}\} \models q$ (no)

   1.3 $\{w_p, w_{pq}\} \models \neg q$ (no)

   1.4 $\{w_p\} \models p ⩖ q$ (yes)

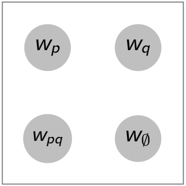
   1.5 $\{w_p, w_q\} \models p ⩖ q$ (no)

   1.6 $\{w_p, w_{pq}\} \models p \rightarrow q$ (no)

2. Show that Inquisitive Logic invalidates LEM: $\not\models \phi ⩖ \neg\phi$

Exercises (Inquisitive logic)

1. Consider the following intensional model:



   Determine whether the following statements are correct:

   1.1 $\{w_p, w_{pq}\} \models p$ (yes)

   1.2 $\{w_p, w_{pq}\} \models q$ (no)

   1.3 $\{w_p, w_{pq}\} \models \neg q$ (no)

   1.4 $\{w_p\} \models p \mathbin{\vee\!\!\!\!\vee} q$ (yes)

   1.5 $\{w_p, w_q\} \models p \mathbin{\vee\!\!\!\!\vee} q$ (no)

   1.6 $\{w_p, w_{pq}\} \models p \rightarrow q$ (no)

2. Show that Inquisitive Logic invalidates LEM: $\not\models \phi \mathbin{\vee\!\!\!\!\vee} \neg\phi$

   $\{w_p, w_q\} \not\models p \mathbin{\vee\!\!\!\!\vee} \neg p$

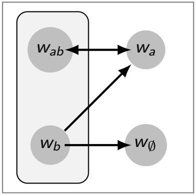
BSML: Classical Modal Logic + NE

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \text{NE} \qquad \text{with } p \in A$$

Bilateral team semantics

Given a Kripke model $M = \langle W, R, V \rangle$ & $s, t, t' \subseteq W$

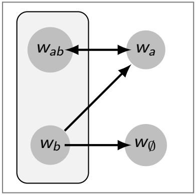


BSML: Classical Modal Logic + NE

Language $\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE}$ with $p \in A$

Bilateral team semantics

Given a Kripke model $M = \langle W, R, V\rangle$ & $s, t, t' \subseteq W$



$M, s \models p$	iff	for all $w \in s : V(w,p) = 1$
$M, s \dashv p$	iff	for all $w \in s : V(w,p) = 0$
$M, s \models \neg\phi$	iff	$M, s \dashv \phi$
$M, s \dashv \neg\phi$	iff	$M, s \models \phi$
$M, s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$
$M, s \dashv \phi \vee \psi$	iff	$M, s \dashv \phi$ & $M, s \dashv \psi$
$M, s \models \phi \wedge \psi$	iff	$M, s \models \phi$ & $M, s \models \psi$
$M, s \dashv \phi \wedge \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \dashv \phi$ & $M, t' \dashv \psi$
$M, s \models \Diamond\phi$	iff	for all $w \in s : \exists t \subseteq R[w] : t \neq \emptyset$ & $M, t \models \phi$
$M, s \dashv \Diamond\phi$	iff	for all $w \in s : M, R[w] \dashv \phi$
$M, s \models \mathrm{NE}$	iff	$s \neq \emptyset$
$M, s \dashv \mathrm{NE}$	iff	$s = \emptyset$

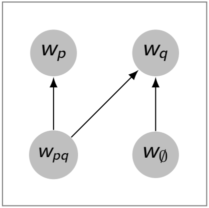
[where $R[w] = \{v \in W \mid wRv\}$]

Logical Consequence $\phi \models \psi$ iff for all M, s: $M, s \models \phi \Rightarrow M, s \models \psi$

Proof Theory. See Anttila 2021; Anttila et al. 2024.

Exercise (BSML)

1. Consider the following Kripke model:

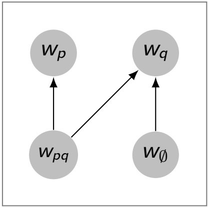


Determine whether the following statements are correct:

1.1 $\{w_p\} \models p \vee q$

Exercise (BSML)

1. Consider the following Kripke model:



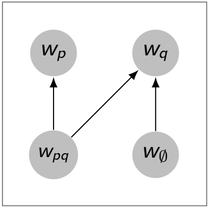
Determine whether the following statements are correct:

1.1 $\{w_p\} \models p \vee q$ (yes)

1.2 $\{w_p\} \models p \vee (q \wedge \textsc{ne})$

Exercise (BSML)

1. Consider the following Kripke model:



Determine whether the following statements are correct:

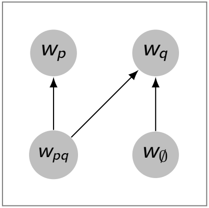
1.1 $\{w_p\} \models p \vee q$ (yes)

1.2 $\{w_p\} \models p \vee (q \wedge \text{NE})$ (no)

1.3 $\{w_p, w_q\} \models p \vee q$

Exercise (BSML)

1. Consider the following Kripke model:



Determine whether the following statements are correct:

1.1 $\{w_p\} \models p \vee q$ (yes)

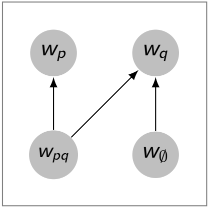
1.2 $\{w_p\} \models p \vee (q \wedge \text{NE})$ (no)

1.3 $\{w_p, w_q\} \models p \vee q$ (yes)

1.4 $\{w_p, w_q\} \models (p \wedge \text{NE}) \vee (q \wedge \text{NE})$

Exercise (BSML)

1. Consider the following Kripke model:



Determine whether the following statements are correct:

1.1 $\{w_p\} \models p \vee q$ (yes)

1.2 $\{w_p\} \models p \vee (q \wedge \mathrm{NE})$ (no)

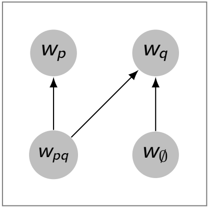
1.3 $\{w_p, w_q\} \models p \vee q$ (yes)

1.4 $\{w_p, w_q\} \models (p \wedge \mathrm{NE}) \vee (q \wedge \mathrm{NE})$ (yes)

1.5 $\{w_\emptyset\} \models \Diamond(p \vee q)$

Exercise (BSML)

1. Consider the following Kripke model:



Determine whether the following statements are correct:

1.1 $\{w_p\} \models p \vee q$ (yes)

1.2 $\{w_p\} \models p \vee (q \wedge \text{NE})$ (no)

1.3 $\{w_p, w_q\} \models p \vee q$ (yes)

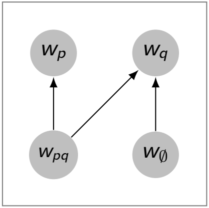
1.4 $\{w_p, w_q\} \models (p \wedge \text{NE}) \vee (q \wedge \text{NE})$ (yes)

1.5 $\{w_\emptyset\} \models \Diamond(p \vee q)$ (yes)

1.6 $\{w_\emptyset\} \models \Diamond((p \wedge \text{NE}) \vee (q \wedge \text{NE}))$

Exercise (BSML)

1. Consider the following Kripke model:



Determine whether the following statements are correct:

1.1 $\{w_p\} \models p \vee q$ (yes)

1.2 $\{w_p\} \models p \vee (q \wedge \mathrm{NE})$ (no)

1.3 $\{w_p, w_q\} \models p \vee q$ (yes)

1.4 $\{w_p, w_q\} \models (p \wedge \mathrm{NE}) \vee (q \wedge \mathrm{NE})$ (yes)

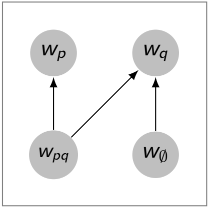
1.5 $\{w_\emptyset\} \models \Diamond(p \vee q)$ (yes)

1.6 $\{w_\emptyset\} \models \Diamond((p \wedge \mathrm{NE}) \vee (q \wedge \mathrm{NE}))$ (no)

1.7 $\{w_{pq}\} \models \Diamond((p \wedge \mathrm{NE}) \vee (q \wedge \mathrm{NE}))$

Exercise (BSML)

1. Consider the following Kripke model:



Determine whether the following statements are correct:

1.1 $\{w_p\} \models p \vee q$ (yes)

1.2 $\{w_p\} \models p \vee (q \wedge \text{NE})$ (no)

1.3 $\{w_p, w_q\} \models p \vee q$ (yes)

1.4 $\{w_p, w_q\} \models (p \wedge \text{NE}) \vee (q \wedge \text{NE})$ (yes)

1.5 $\{w_\emptyset\} \models \Diamond(p \vee q)$ (yes)

1.6 $\{w_\emptyset\} \models \Diamond((p \wedge \text{NE}) \vee (q \wedge \text{NE}))$ (no)

1.7 $\{w_{pq}\} \models \Diamond((p \wedge \text{NE}) \vee (q \wedge \text{NE}))$ (yes)

2. Does BSML validate LEM $\phi \vee \neg\phi$?

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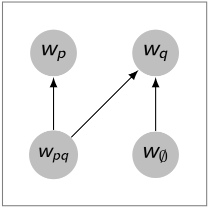

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2. Does BSML validate LEM $\phi \vee \neg\phi$?

No, $\{w_p\} \not\models ((p \wedge \text{NE}) \vee (q \wedge \text{NE})) \vee \neg((p \wedge \text{NE}) \vee (q \wedge \text{NE}))$

Exercises (bonus)

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   DL: $T_1 \models dep(x, y)$, $T_2 \models dep(x, y)$, but $T_1 \cup T_2 \not\models dep(x, y)$

T_1	x	y
i_1	0	0

T_2	x	y
i_2	0	1

2. Show that BSML is union closed

   Proof by induction, see Anttila (2021), Proposition 2.2.8, page 21, `https://eprints.illc.uva.nl/id/eprint/1788/1/MoL-2021-03.text.pdf`

3. Consider the following new closure property

   Convexity
   ϕ is convex if $[s \models \phi, t \models \phi$ and $s \subseteq u \subseteq t]$ implies $u \models \phi$

   Check whether convexity holds for all formulas in Dependence Logic, Inquisitive Logic and BSML.

   Convexity holds for all formulas in all 3 systems: in Dependence Logic & Inquisitive Logic because it is a consequence of downward closure; in BSML because it is a consequence of union closure.

qBSML: Quantified Modal Logic + NE

Language:

$t ::= c|v$

$\phi ::= P^n(\vec{t}) \mid \phi \vee \psi \mid \phi \wedge \psi \mid \exists v\phi \mid \forall v\phi \mid \Box\phi \mid$ NE

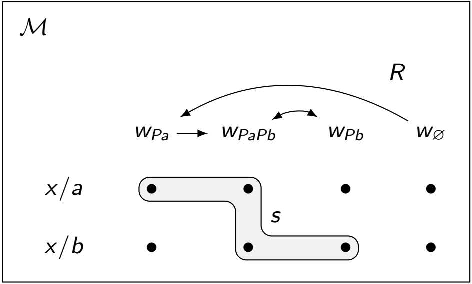
Model:

$\mathcal{M} = \langle W, D, R, I\rangle$

Information State:

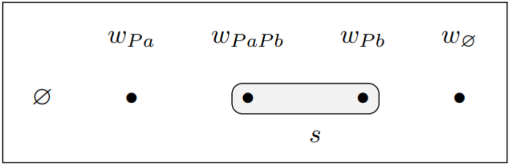
A state is set of indices $i = \langle w_i, g_i\rangle$, where $w_i \in W$ and g_i is a variable assignment function

Example:



$$s = \{\langle w_{Pa}, g[x/a]\rangle, \langle w_{PaPb}, g[x/a]\rangle, \langle w_{PaPb}, g[x/b]\rangle, \langle w_{Pb}, g[x/b]\rangle\}$$

Empty assignment



A state with an empty assignment

What happens when a variable is added to such information state?

Operations on States

***x*-extension of an assignment:**

$$g[x/d] := (g \setminus \{\langle x, g(x)\rangle\}) \cup \{\langle x, d\rangle\}$$

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Individual x-extension of an index:

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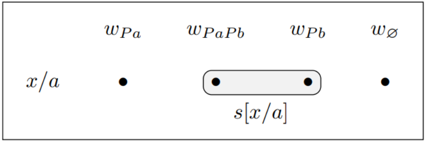
Individual x-extension of a state:

$$s[x/d] := \{i[x/d] | i \in s\}$$

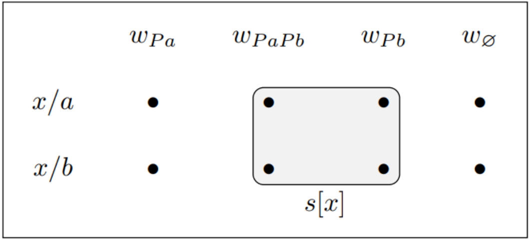


Individual x-extension

Operations on States

Universal *x*-extension:

$$s[x] := \{i[x/d] | i \in s \;\&\; d \in D\}$$



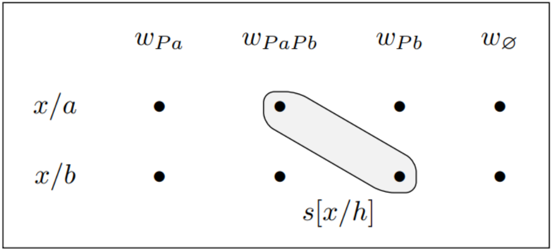
Universal *x*-extension

Assume $D = \{a, b\}$

Operations on States

Functional *x*-extension:

$$s[x/h] := \{i[x/d] | i \in s \ \& \ d \in h(i)\}$$
$$h : s \mapsto \wp(D) \backslash \varnothing$$



Functional *x*-extension

Semantic Clauses

$$\mathcal{M}, s \models P^n t_1 \dots t_n \quad \text{iff} \quad \forall i \in s : \langle [t_1]_{\mathcal{M},i}, \dots, [t_n]_{\mathcal{M},i} \rangle \in I(w_i)(P^n)$$
$$\mathcal{M}, s \dashv P^n t_1 \dots t_n \quad \text{iff} \quad \forall i \in s : \langle [t_1]_{\mathcal{M},i}, \dots, [t_n]_{\mathcal{M},i} \rangle \notin I(w_i)(P^n)$$
$$\mathcal{M}, s \models \neg\varphi \quad \text{iff} \quad \mathcal{M}, s \dashv \varphi$$
$$\mathcal{M}, s \dashv \neg\varphi \quad \text{iff} \quad \mathcal{M}, s \models \varphi$$
$$\mathcal{M}, s \models \varphi \vee \psi \quad \text{iff} \quad \exists t, t' : t \cup t' = s \text{ and } \mathcal{M}, t \models \varphi \text{ and } \mathcal{M}, t' \models \psi$$
$$\mathcal{M}, s \dashv \varphi \vee \psi \quad \text{iff} \quad \mathcal{M}, s \dashv \varphi \text{ and } \mathcal{M}, s \dashv \psi$$
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$$\mathcal{M}, s \models \Box\varphi \quad \text{iff} \quad \forall i \in s : \mathcal{M}, R(w_i)[g_i] \models \varphi$$
$$\mathcal{M}, s \dashv \Box\varphi \quad \text{iff} \quad \forall i \in s : \exists X \subseteq R(w_i) \text{ and } X \neq \emptyset \text{ and } \mathcal{M}, X[g_i] \dashv \varphi$$
$$\mathcal{M}, s \models \mathtt{NE} \quad \text{iff} \quad s \neq \emptyset \qquad [X[g_i] = \{\langle w, g_i \rangle \mid w \in X\}]$$
$$\mathcal{M}, s \dashv \mathtt{NE} \quad \text{iff} \quad s = \emptyset \qquad [R(w_i) = \{v \in W \mid w_i R v\}]$$
$$\mathcal{M}, s \models \forall x\varphi \quad \text{iff} \quad \mathcal{M}, s[x] \models \varphi$$
$$\mathcal{M}, s \dashv \forall x\varphi \quad \text{iff} \quad \mathcal{M}, s[x/h] \dashv \varphi, \text{ for some } h : s \to \wp(D) \setminus \emptyset$$
$$\mathcal{M}, s \models \exists x\varphi \quad \text{iff} \quad \mathcal{M}, s[x/h] \models \varphi, \text{ for some } h : s \to \wp(D) \setminus \emptyset$$
$$\mathcal{M}, s \dashv \exists x\varphi \quad \text{iff} \quad \mathcal{M}, s[x] \dashv \varphi$$

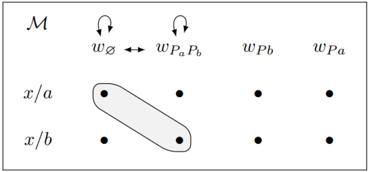
Illustration: modality

$M, s \models \Box\phi \iff \forall i \in s : R(w_i)[g_i] \models \phi$
$M, s \models \Diamond\phi \iff \forall i \in s : \exists X \subseteq R(w_i)$ and $X \neq \varnothing$ and $X[g_i] \models \phi$

$X[g_i] = \{\langle w, g_i\rangle \mid w \in X\}$ & $R(w_i) = \{v \in W \mid w_i R v\}$.

$s^{\downarrow} := \{w \in W | \langle w, g\rangle \in s\}$

R is **state-based** in (M, s) iff $\forall w \in s^{\downarrow} : R(w) = s^{\downarrow}$
R is **indisputable** (M, s) iff $\forall w, v \in s^{\downarrow} : R(w) = R(v)$



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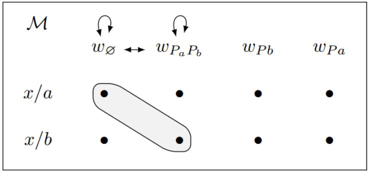

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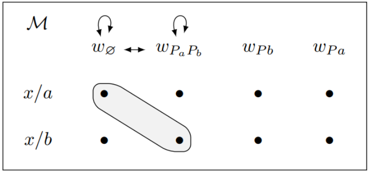

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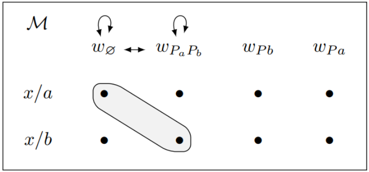

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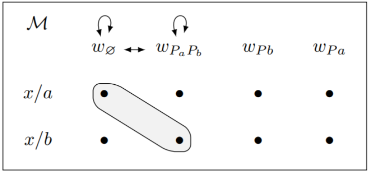

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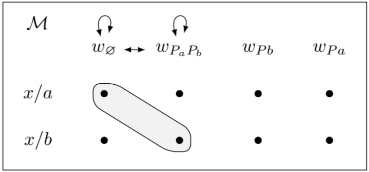

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Modified Numerals

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$$\text{at least } n(A, B) \text{ iff } A \cap B > n-1$$

$$\text{more than } n-1(A, B) \text{ iff } A \cap B > n-1$$

But (b) examples below carry an **ignorance inference**.

(5) a. The house has more than two bedrooms.
b. The house has at least three bedrooms.

(6) a. A pentagon has more than 3 sides.
b. ?A pentagon has at least 4 sides.

(7) a. I have more than 2 children.
b. ?I have at least 3 children.

Modified Numerals

Büring (2008) proposes that superlative modified numerals involve **disjunctive meanings**:

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Modified numerals display a number of different effects including obviation under universal quantifiers, distribution effects and cancellation under negation.

These effects can all be captured in qBSML by assuming the above lexical entry and the results carry over given the operation of neglect-zero enrichment on disjunction we saw yesterday.

qBSML and Modified Numerals

Klaus has at least three children. [Ignorance]
$(\texttt{three} \vee \texttt{more})^+ \models \Diamond\texttt{three} \wedge \Diamond\texttt{more}$

Every woman in my family has at least three children. [Obviation]
$(\forall x(\texttt{three}(x) \vee \texttt{more}(x)))^+ \not\models \forall x(\Diamond\texttt{three}(x) \wedge \Diamond\texttt{more}(x))$

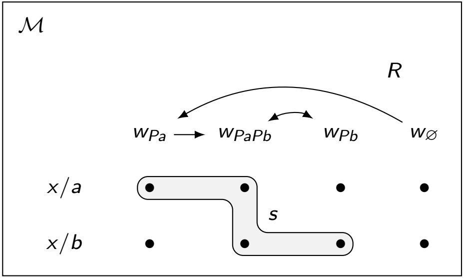
Every woman in my family has at least three children. [Distribution$^\Diamond$]
$(\forall x(\texttt{three}(x) \vee \texttt{more}(x)))^+ \models \exists x\Diamond\, \texttt{three}(x) \wedge \exists x\Diamond\, \texttt{more}(x)$

You are required to read at least three books. [$\Box$ free choice]
$(\Box(\texttt{three} \vee \texttt{more}))^+ \models \Diamond\, \texttt{three} \wedge \Diamond\, \texttt{more}$

You are allowed to read at least three books. [$\Diamond$ free choice]
$(\Diamond(\texttt{three} \vee \texttt{more}))^+ \models \Diamond\, \texttt{three} \wedge \Diamond\, \texttt{more}$

?Klaus does not have at least three children. [Negation]
$(\neg(\texttt{three} \vee \texttt{more}))^+ \models \neg\texttt{three} \wedge \neg\texttt{more}$

Exercise



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