Philosophical and linguistic applications of team semantics

Class 4: Inquisitive Semantics

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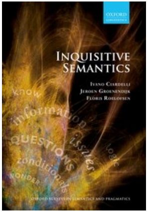
Tsinghua Logic Summer School 2024

Inquisitive semantics

A team semantics for the formal analysis of information exchange

Main goal: capture both informative and inquisitive aspects of meaning



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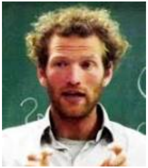
Linguistic Motivations

- Capture the meaning of questions (ch. 4)
- Represent inquisitive attitude verbs and their selective properties (ch. 8)

(a) Groenendijk



(b) Roelofsen

(c) Ciardelli

Three perspectives on the notion of a question

- **Syntax**: sentence with distinctive features: word order, *wh*-pronouns,...
  $\mapsto$ *interrogative sentence*

(1) a. Is it raining? [polar (*yes-no*-)interrogative]
    b. Who called? [constituent (*wh*-)interrogative]

- **Pragmatics**: act performed by uttering an interrogative sentence: request to provide information $\mapsto$ *interrogative act*

(2) a. Why is there something rather than nothing?
    b. Please, tell me why there is something rather than nothing. [imperative with embedded interrogative]

- **Semantics**: abstract object denoted by an interrogative sentence, or content of an interrogative act $\mapsto$ *question*

What interrogatives mean

- Classical view on meaning (Wittgenstein TLP):
  - Semantic content of a sentence = its truth conditions
  - To know the meaning of a sentence is to know the conditions under which the sentence is true
- Implementation in classical possible world semantics:
  - Meanings specified via truth-conditions: $M, w \models \phi$
  - Sentences express classical propositions: $|\phi|_M = \{w \in W_M \mid M, w \models \phi\}$
- But interrogatives (or imperatives) lack truth conditions:

(3) a. Who closed that door?
    b. Kill Bill!

Meaning of non-declarative sentence types

One influential account (Hamblin)

- Interrogatives
  - Semantic content of an interrogative = its answerhood conditions
  - To know the meaning of an interrogative is to know what counts as an answer to the expressed question.
- Imperatives
  - Semantic content of an imperative = its compliance conditions
  - To know the meaning of an imperative is to know what has to be true for the command expressed by the imperative to be complied with.

Meaning of non-declarative sentence types

Alternative views

- Frege, Stenius 1964, Searle 1969: thought/sentence radical + force/mood

(4) You are late. Are you late? Be late!

a. Common sentence-radical: *that addressee is late*
b. Differing moods: declarative, interrogative and imperative

- Lewis 1970: Non-declaratives as paraphrases of corresponding performatives:

(5) a. Are you late? ≡ I ask you whether you are late.
b. Be late! ≡ I command you to be late.

Spurious truth values: always true when uttered

- Dynamic approaches (Stalnaker 1978, Portner 2005, Aloni et al 2007[1], Krifka 2022, 2024): different moods correspond to different ways to update a common ground

[1] Aloni, M, Beaver, D, Clark, B, and Rooij, R van. 2007. "The dynamics of topics and focus." In *Questions in Dynamic Semantics*. Eds. M. Aloni, P. Dekker, and A. Butler. CRiSPI.

Back to Hamblin tradition

Defining questions in terms of their answers

- Some answer types:

(6) Who called?

a.	John called	[possible (true) propositional answer]
b.	Only John called	[strong exhaustive answer]
c.	John	[constituent/term answer]
d.	#The sun is shining today	[not an answer]

- Different frameworks take different answer types as central:
  - **Alternative set theories** (Hamblin/Karttunnen):

    $\mapsto$ POSSIBLE (TRUE) ANSWER
  - **Partition theory** (Groenendijk & Stokhof):

    $\mapsto$ EXHAUSTIVE ANSWER
  - **Categorial/Structured meaning theories** (e.g. Ajdukiewicz, vStechow, Krifka, Ginzburg):

    $\mapsto$ CONSTITUENT ANSWER

Inquisitive semantics view

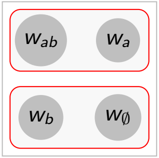
Interrogatives (standard view)

- To know the meaning of an interrogative sentence is to know what counts as an answer.
- Interrogative meaning = answerhood conditions.

Interrogatives (inquisitive view)

- To know the meaning of an interrogative sentence is to know what information is needed to resolve it.
- Interrogative meaning = resolution conditions.

Answerhood vs resolution conditions



- Illustration:

(7) Is Aicha coming?

(8) a. Yes, Aicha is coming $\mapsto \{w_a, w_{ab}\}$ (basic answer)
b. No, Aicha is not coming $\mapsto \{w_b, w_\emptyset\}$ (basic answer)
c. Yes, Aicha and Bo are coming $\mapsto \{w_{ab}\}$ (resolving info)
d. No, only Bo is coming $\mapsto \{w_b\}$ (resolving info)

- Answerhood conditions: a set of classical propositions which constitute basic answers to the question

  $\mapsto \{\{w_a, w_{ab}\}, \{w_b, w_\emptyset\}\}$

- Resolution conditions: a downward closed set of information states, all the info states which resolve the issue raised by the interrogative

  $\mapsto \{\{w_a, w_{ab}\}, \{w_b, w_\emptyset\}, \{w_a\}, \{w_{ab}\}, \{w_b\}, \{w_\emptyset\}, \emptyset\}$

  (but not including, e.g., $\{w_a, w_b\}$, which does not resolve the issue)

- Main motivation: a uniform account of entailment and logical operations which apply to both declaratives and interrogatives

Guidelines from Groenendijk and Stokhof

G&S on explanatory adequacy

> *[. . .] It seems natural to require of a semantic theory that deals with a certain domain of phenomena, that it account for such phenomena as they occur elsewhere too, by using general principles, notions and operations which can be applied outside the particular domain of the theory as well.*
> [*Studies on the semantics of questions and the pragmatics of answers, p. 11]*

- In general, explanatory power gained by adopting general principles or operations which can be applied across different domains
- Here: explanatory adequacy requires general notions which apply to both declaratives and interrogatives:
  - A general notion of entailment defined in terms of set inclusion;
  - A uniform account of logical operations: conjunction uniformly interpreted as intersection; disjunction as set union; etc
- Inquisitive argument (Ciardelli, Groenendijk & Roelofsen):
  - Hamblin's answerhood conditions are not adequate to meet G&S's requirements, while inquisitive resolutions are;
  - (Also G&S partitions would satisfy these requirements. But at the end, inquisitive resolutions are more general and empirically more adequate, e.g. mention-some questions).

Answerhood vs resolution conditions: Coordinated interrogatives

(9) a. Is Aicha coming?
  b. Is Bo coming?
  c. Is Aicha coming? And, is Bo coming?

To satisfy G&S's requirements, we need: $|(9\text{-c})| = |(9\text{-a})| \cap |(9\text{-b})|$

With answerhood conditions, we can't have it. We need resolutions:

(10) Is Aicha coming? $\mapsto$ 

  a. Answerhood: $\{\{w_a, w_{ab}\}, \{w_b, w_\emptyset\}\}$
  b. Resolution: $\{\{w_a, w_{ab}\}, \{w_a\}, \{w_{ab}\}, \{w_b, w_\emptyset\}, \{w_b\}, \{w_\emptyset\}, \emptyset\}$

(11) Is Bo coming? $\mapsto$ 

  a. Answerhood: $\{\{w_b, w_{ab}\}, \{w_a, w_\emptyset\}\}$
  b. Resolution: $\{\{w_b, w_{ab}\}, \{w_b\}, \{w_{ab}\}, \{w_a, w_\emptyset\}, \{w_a\}, \{w_\emptyset\}, \emptyset\}$

(12) Is Aicha coming? And, is Bo coming? $\mapsto$ 

  a. $\{\{w_a, w_{ab}\}, \{w_b, w_\emptyset\}\} \cap \{\{w_b, w_{ab}\}, \{w_a, w_\emptyset\}\} = \emptyset$ $\Leftarrow$ problem!
  b. $\{\{w_a, w_{ab}\}, \{w_a\}, \{w_{ab}\}, \{w_b, w_\emptyset\}, \{w_b\}, \{w_\emptyset\}, \emptyset\} \cap$
  $\{\{w_b, w_{ab}\}, \{w_b\}, \{w_{ab}\}, \{w_a, w_\emptyset\}, \{w_a\}, \{w_\emptyset\}, \emptyset\} =$
  $\{\{w_a\}, \{w_b\}, \{w_{ab}\}, \{w_\emptyset\}, \emptyset\}$

Answerhood vs resolution conditions: interrogative entailment

"It seems natural to consider one interrogative entailing another as every proposition giving an answer to the first gives an answer to the second."

For instance, (13-a) entails (13-b), which in turn entails (13-c).

(13) a. Who is coming and who went to the cinema?
b. Who is coming?
c. Is Aicha coming?

To satisfy G&S's requirement, we need: $|(13\text{-a})| \subseteq |(13\text{-b})| \subseteq |(13\text{-c})|$

(14) Is Aicha coming? $\mapsto$ 

a. Answerhood: $\{\{w_a, w_{ab}\}, \{w_b, w_\emptyset\}\}$
b. Resolution: $\{\{w_a, w_{ab}\}, \{w_a\}, \{w_{ab}\}, \{w_b, w_\emptyset\}, \{w_b\}, \{w_\emptyset\}, \emptyset\}$

(15) Who is coming?

a. Answerhood (Hamblin): $\{\{w_a, w_{ab}\}, \{w_b, w_{ab}\}\} \mapsto$ 
b. Resolution (partition): $\{\{w_a\}, \{w_b\}, \{w_{ab}\}, \{w_\emptyset\}, \emptyset\} \mapsto$ 

But $|(15\text{-a})| \not\subseteq |(14\text{-a})|$, while $|(15\text{-b})| \subseteq |(14\text{-b})|$

Summary desiderata: Generalised entailment

A general notion of entailment defined in terms of set inclusion

1. Between declaratives

   (16) Bo and Aicha went to the party $\Rightarrow$ Aicha went to the party

2. Between interrogatives

   (17) Who went to the party? $\Rightarrow$ Did Aicha go to the party?

3. In between declaratives and interrogatives

   (18) Aicha went to the party $\Rightarrow$ Who went to the party?

Summary desiderata: Logical operations

A uniform account of logical operations that can be performed on declaratives and interrogatives

(19) a. Alice rented a car and she booked a hotel.
b. Did Alice rent a car and did she book an hotel?
c. What car did Alice rent, and which hotel did she book?

(20) a. Alice rented a car or she took a train.
b. Did Alice rent a car or did she take a train?
c. What car did Alice rent or which train did she take?

(21) a. If Aicha wins a free trip, she'll go to Beijjin.
b. If Aicha wins a free trip, will she accept it?
c. If Aicha wins a free trip, where will she go?

(22) a. Bob knows that Alice lives in Galway.
b. Bob knows whether Alice lives in Galway.
c. Bob knows where Alice lives.

Summary desiderata: More on attitude verbs

Entailment patterns & selection properties of inquisitive attitude verbs

(23) It is raining & Aicha knows whether it is raining ⇒ Aicha knows that it is raining

(24) It is not raining & Aicha knows whether it is raining ⇒ Aicha knows that it is not raining

(25) a. Aicha knows that it is raining
b. Aicha knows whether it is raining

(26) a. Aicha #wonders that it is raining
b. Aicha wonders whether it is raining

(27) a. Aicha believes that it is raining
b. Aicha #believes whether it is raining

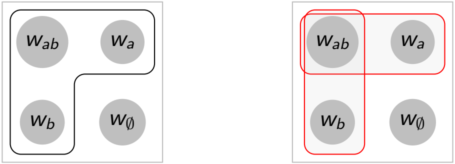
Inquisitive Semantics

- Formulas are not evaluated with respect to states of affair (possible worlds) but wrt states of information (sets of possible worlds);
- Both declarative and interrogative sentences express inq-propositions Π, sets of information states;
  - Classical propositions: sets of possible worlds

$$|\phi|^C_M = \{w \in W \mid M, w \models \phi\}$$

  - Inq-propositions: sets of information states

$$|\phi|^I_M = \{s \subseteq W \mid M, s \models \phi\}$$



(d) classical (e) inquisitive

Figure: Classical vs Inquisitive

Inquisitive Logic: CPL + $\vee\!\!\!/$

Language

[Ciardelli & Roelofsen 2011]

$$\phi := p \mid \bot \mid \phi \rightarrow \phi \mid \phi \wedge \phi \mid \phi \vee\!\!\!/\ \phi$$

Defined connectives: $\neg\phi := \phi \rightarrow \bot$ & $\phi \vee_* \psi := \neg(\neg\phi \wedge \neg\psi)$

Team/state-based semantics

Given a model $M = (W, V)$ and an information state $s \subseteq W$,

- $M, s \models p \Leftrightarrow$ for all $w \in s : V(p, w) = 1$
- $M, s \models \bot \Leftrightarrow s = \emptyset$
- $M, s \models \phi \wedge \psi \Leftrightarrow M, s \models \phi$ and $M, s \models \psi$
- $M, s \models \phi \rightarrow \psi \Leftrightarrow$ for all $t \subseteq s : M, t \models \phi$ implies $M, t \models \psi$
- $M, s \models \phi \vee\!\!\!/\ \psi \Leftrightarrow M, s \models \phi$ or $M, s \models \psi$

Defined connectives

- $M, s \models \neg\phi \Leftrightarrow$ for all $t \subseteq s$ $M, t \not\models \phi$, unless $t = \emptyset$
- $M, s \models \phi \vee_* \psi \Leftrightarrow$ for all $w \in s : M, \{w\} \models \phi$ or $M, \{w\} \models \psi$

Properties of inq-propositions

- Inq-propositions not only convey information, but also express issues;
  - The informative content of Π is given by the info state $\bigcup(\Pi)$=info(Π)
  - The issue expressed by Π is the one which is resolved by some s iff $s \in \Pi$

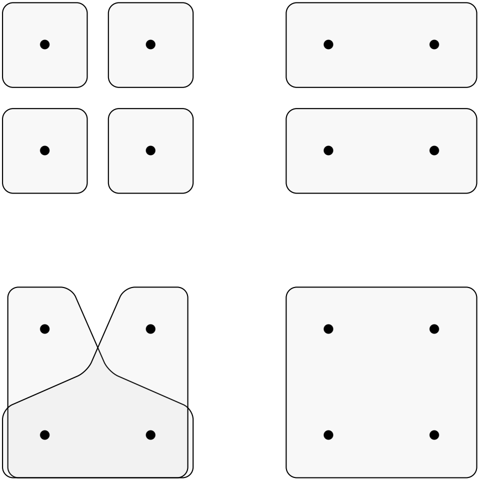
Π:  & info(Π): 

- Two properties of inq-propositions:
  1. downward closed: if s resolves Π, then any $t \subseteq s$ resolves Π as well

$$s \in \Pi \;\&\; t \subseteq s \;\Rightarrow\; t \in \Pi$$

  2. non-empty, because the absurd info state $\emptyset$ is assumed to resolve any issue.
- Two examples of inq-propositions (only alternatives represented)
  - The **Trivial** Π: $\wp(W)$ 
  - The **Absurd** Π: $\{\emptyset\}$ 
- Maximal elements of an inq-proposition are called **alternatives**

Examples of inq-propositions (only alternatives represented)

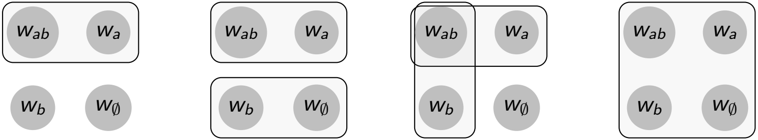


Informativeness and inquisitiveness

Π is **informative** iff $\text{info}(\Pi) \neq W$ (does not cover the logical space)

Π is **inquisitive** iff $\text{info}(\Pi) \notin \Pi$ (has more than one alternative)

Examples:



(a) +info −inqui (b) −info +inqui (c) +info +inqui (d) −info −inqui

Figure: Informative vs Inquisitive

Which formulas express these inq-propositions (only alt represented)?

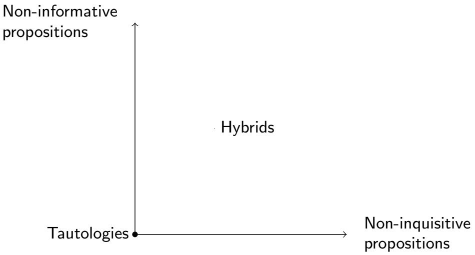
$$\text{(a): } a; \quad \text{(b): } a \vee\!\!\!/\ \neg a; \quad \text{(c): } a \vee\!\!\!/\ b; \quad \text{(d): } \neg\bot.$$

(28) Aicha will attend $\mapsto a$ $\quad |a|^I = \{\{w_a, w_{ab}\}, \{w_a\}, \{w_{ab}\}, \emptyset\}$

(29) Will Aicha attend? $\mapsto a \vee\!\!\!/\ \neg a$ $\quad |a \vee\!\!\!/\ \neg a|^I = \{\{w_a, w_{ab}\}, \{w_a\}, \{w_{ab}\}, \{w_b, w_\emptyset\}, \{w_b\}, \{w_\emptyset\}, \emptyset\}$

Four categories

statements:	non-inquisitive	$\text{info}(\Pi) \in \Pi$	$\Leftrightarrow \Pi = \wp(\text{info}(\Pi))$
questions:	non-informative	$\text{info}(\Pi) = W$	
tautologies:	neither	$\Pi = \wp(W)$	
hybrids:	both		



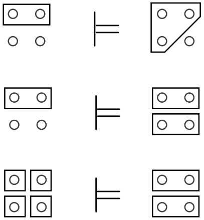
Entailment

$\phi \models \psi$ iff for all M, s: $M, s \models \phi$ implies $M, s \models \psi$

1. $|\phi|$ is **at least as informative** as $|\psi|$: $\text{info}(|\phi|) \subseteq \text{info}(|\psi|)$
2. $|\phi|$ is **at least as inquisitive** as $|\psi|$: $|\phi| \subseteq |\psi|$

iff $|\phi| \subseteq |\psi|$

Examples:



Some examples

$$?\phi := \phi \vee\!\!\!/ \neg\phi$$

(30) Aicha will attend the meeting↓.  α

(31) Aicha or Bo will attend the meeting↓.  $\alpha \vee_* \beta$

(32) Will Aicha attend?  $?\alpha$

(33) Will Aicha-or-Bo attend?  $?(\alpha \vee_* \beta)$

(34) Will Aicha↑ attend, or Bo↑?  $?(\alpha \vee\!\!\!/ \beta)$

(35) If Aicha attends, will Bo attend?  $\alpha \rightarrow ?\beta$

(36) Who will attend?   $?\exists x\phi$; $\forall x?\phi(x)$

Adding quantifiers (simplifying)

Predication: $M, s \models Rt_1, \ldots, t_n$ iff $s \subseteq |Rt_1, \ldots, t_n|^C_M$

Quantification:

$\exists x Rx \equiv Ra \mathbin{\vee\!\!\!/} Rb$ — assume $D = \{a, b\}$

$\forall x Rx \equiv Ra \wedge Rb$

$\exists_* x Rx \equiv Ra \vee_* Rb$

$\forall x ?Rx \equiv \forall x (Rx \mathbin{\vee\!\!\!/} \neg Rx)$ — mention-all question

$?\exists x Rx \equiv \exists x Rx \mathbin{\vee\!\!\!/} \neg \exists x Rx$ — mention-some question

$?\forall x Rx \equiv \forall x Rx \mathbin{\vee\!\!\!/} \neg \forall x Rx$ — polar question all

$?\exists_* x Rx \equiv \exists_* x Rx \mathbin{\vee\!\!\!/} \neg \exists_* x Rx$ — polar question some

Examples:

(37) Who will attend?   $?\exists x Rx$; $\forall x ?Rx$

(38) Where can I buy an Italian newspaper?  $?\exists x Rx$

(39) Has anybody arrived?  $?\exists_* x Rx$

(40) Is everybody happy? $?\forall x Rx$

Inquisitive Epistemic Logic: Overview

Clause-embedding predicates can be:

- Information-directed, e.g., *know* and *believe*
- Issue-directed, e.g., *wonder* and *be curious*

How to represent their meanings within the framework of inquisitive semantics?

- Standard epistemic logic and its limitations
- Inquisitive Epistemic Logic
- Inquisitive treatment of *know*
- Inquisitive treatment of *wonder*

Standard epistemic logic

- The language has a modal operator K_a for each agent
- In every possible world w, every agent a is associated with an information state $\sigma_a(w)$, the epistemic state of a at w.
  - This is equivalent to the relational treatment ($\sigma_a(w) = \{v \in W \mid wR_a v\}$)
  - We can impose further constraints on σ_a, e.g., introspection
- $w \models K_a\varphi \quad \Leftrightarrow \quad \sigma_a(w) \subseteq |\varphi|^C$

  where $|\varphi|^C$ is the set of worlds where φ is true
- At an abstract level, K_a compares two sets of worlds, i.e., is a relation between two classical propositions

Limitations of the standard account

Limitation 1: interrogative complements of *know*

(41) Alice knows whether Bob is coming.

(42) Alice knows whether Bob or Charlie is coming.

(43) Alice knows who is coming.

Classical analyses attempt to reduce these cases to ones with declarative complements

- Specifically, the declarative complement is assumed to be the complete true answer

Limitations of the standard account

Problem: mention-some readings

(44) Alice knows where one can buy an Italian newspaper in Amsterdam.

This is not reducible to Alice knowing a particular classical proposition.

Limitations of the standard account

Limitation 2: issue-directed attitudes, e.g., *wonder*, *be curious*

Embedded interrogatives not reducible to embedded declaratives

(45) Alice wonders whether Bob is coming.

(46) #Alice wonders that Bob is coming.

No direct account of the entailment involving *wonder* and *know*

(47) Alice wonders who the culprit is.

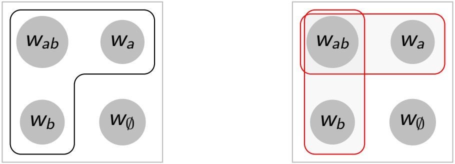
(48) Alice knows that the culprit is Bob or Charlie.

(49) So, Alice wonders whether the culprit is Bob or Charlie.

The inquisitive approach

A uniform treatment of declarative and interrogative complements

- Classical picture: attitudes are relations between two sets of worlds (an information state and a classical proposition)
- Attitudes are treated as relations between two sets of information states, i.e., an inquisitive state and an inq-proposition



(a) classical (b) inquisitive

Figure: Classical vs inquisitive state

Inquisitive Epistemic Logic

Language

[Ciardelli 2016]

$$\phi ::= p \mid \bot \mid \phi \to \phi \mid \phi \wedge \phi \mid \phi \vee\!\!\!/\, \phi \mid \Box\phi \mid \boxplus\, \phi \qquad \text{where } p \in PROP$$

Defined connectives: $\neg\phi := \phi \to \bot$

Model

A model is a triple $M = (W, \Sigma, V)$ where

- W is a non-empty set of possible worlds;
- Σ is a function which assigns an inq-state $\Sigma(w)$ to each $w \in W$;
- $V : W \times PROP \to \{0, 1\}$ is a valuation function.

Notation: We write $\sigma(w) := \bigcup(\Sigma(w))$

$\mapsto$ The classical epistemic state $\sigma(w)$ can be retrieved from the inquisitive state $\Sigma(w)$

Inquisitive Epistemic Logic

State-based semantics

Given a model $M = (W, \Sigma, V)$ and an information state $s \subseteq W$,

- $M, s \models p \Leftrightarrow$ for all $w \in s : V(w, p) = 1$
- $M, s \models \bot \Leftrightarrow s = \emptyset$
- $M, s \models \phi \wedge \psi \Leftrightarrow M, s \models \phi$ and $M, s \models \psi$
- $M, s \models \phi \to \psi \Leftrightarrow$ for all $t \subseteq s : t \models \phi$ implies $t \models \psi$
- $M, s \models \phi \mathbin{\vee\!\!\!/} \psi \Leftrightarrow M, s \models \phi$ or $M, s \models \psi$
- $M, s \models \Box\phi \Leftrightarrow$ for all $w \in s : M, \sigma(w) \models \phi$ (= BSML/BSEL)
- $M, s \models \boxplus\phi \Leftrightarrow$ for all $w \in s :$ for all $t \in \Sigma(w) : M, t \models \phi$

Epistemic interpretation of the modals:

- $\Box\phi$: the agent's information state resolves ϕ
- $\boxplus\phi$: the agent entertains the issue ϕ

Know $\mapsto \Box$ ($\neq$ BSEL); Wonder $\mapsto \boxplus$ & $\neg\Box$.

Inquisitive Epistemic Logic

Definition
A formula α is flat / truth-conditional iff for every M, s:

$$M, s \models \alpha \Leftrightarrow M, \{w\} \models \alpha, \text{for all } w \in s$$

Definition
A formula ϕ is declarative if every occurrence of $\mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}}$ in ϕ is in the scope of some modal operator $\Box, \boxplus$.

Fact (Ciardelli 2016) Every declarative formula is truth-conditional

For declarative formulas we can define truth-conditions:

- $M, w \models \phi$ iff $M, \{w\} \models \phi$

Easy to see that if ϕ is truth-conditional then $|\phi|^I$ is non-inquisitive (only one alternative).

Know

- $K\phi := \Box\phi$

  with additional constraints on the accessibility relation, e.g., reflexivity, positive and negative introspection

- $\Box\phi$ is declarative/truth-conditional:
  - Support: $M, s \models K\phi \Leftrightarrow$ for all $w \in s : M, \sigma(w) \models \phi$
  - Truth-conditions: $M, w \models K\phi \Leftrightarrow M, \sigma(w) \models \phi$
- An agent knows ϕ in w iff the agent's info state in w, $\sigma(w)$, supports ϕ
- While $\Box\phi$ is truth-conditional, ϕ needs not be (it could be a question)
- If ϕ truth-conditional ($\phi = \alpha$):
  - $M, w \models K\alpha \Leftrightarrow M, \sigma(w) \models \alpha \Leftrightarrow \sigma(w) \subseteq |\phi|^C$ (just as in standard EL)
- If ϕ is a question ($\phi = ?\alpha$ (with α truth-conditional)):
  - $M, w \models K?\alpha \;\Leftrightarrow\; M, \sigma(w) \models \alpha \,\mathbin{\vee\!\!\!/}\, \neg\alpha \;\Leftrightarrow\; M, \sigma(w) \models \alpha$ or $M, \sigma(w) \models \neg\alpha$ $\Leftrightarrow \sigma(w) \subseteq |\alpha|^C$ or $\sigma(w) \subseteq |\neg\alpha|^C$
- Uniform account of knowing-*that* and knowing-*wh* (extending standard El)

Know: examples

(50) Ali knows that it is raining $\mapsto K\alpha$ — $|\alpha|$: 

(51) Ali knows that it is not raining $\mapsto K\neg\alpha$ — $|\neg\alpha|$: 

(52) Ali knows whether it is raining $\mapsto K?\alpha$ — $|?\alpha|$: 

$\sigma(w_1)$:  $\sigma(w_2)$:  $\sigma(w_3)$: 

- What is true in w_1, w_2, w_3? Recall: $M, w \models K\phi \Leftrightarrow M, \sigma(w) \models \phi$
  - $M, w_1 \models K\alpha$, $M, w_1 \not\models K\neg\alpha$, $M, w_1 \models K?\alpha$
  - $M, w_2 \not\models K\alpha$, $M, w_2 \models K\neg\alpha$, $M, w_2 \models K?\alpha$
  - $M, w_3 \not\models K\alpha$, $M, w_3 \not\models K\neg\alpha$, $M, w_3 \not\models K?\alpha$

Know: entailments

(53) Ali knows that it is raining ⇒ Ali knows whether it is raining $\mapsto K\alpha \models K?\alpha$

(54) Ali knows that it is not raining ⇒ Ali knows whether it is raining $\mapsto K\neg\alpha \models K?\alpha$

(55) It is raining & Ali knows whether it is raining ⇒ Ali knows that it is raining $\mapsto \alpha, K?\alpha \models K\alpha$

(56) Ali knows whether it is raining & it is not raining ⇒ Ali knows that it is not raining $\mapsto \neg\alpha, K?\alpha \models K\neg\alpha$

$\neg\alpha, K?\alpha \models K\neg\alpha$

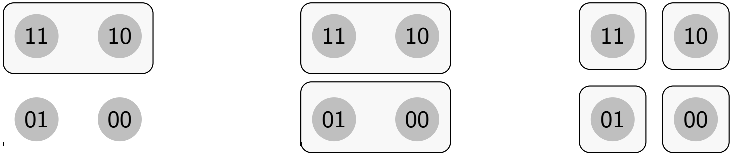
Proof. $M, w \models \neg\alpha \Rightarrow w \notin |\alpha|^C$. Then by reflexivity, $\sigma(w) \not\subseteq |\alpha|^C$. $M, w \models K?\alpha \Rightarrow \sigma(w) \subseteq |\alpha|^C$ or $\sigma(w) \subseteq |\neg\alpha|^C$. Since $\sigma(w) \not\subseteq |\alpha|^C$, it must be $\sigma(w) \subseteq |\neg\alpha|^C$. It follows $M, w \models K\neg\alpha$.

Exercise: know

(57) Alice knows that Bob is coming. KCb

(58) Alice knows whether Bob is coming. $K?Cb$

(59) Alice knows who is coming. $K(\forall x?Cx)$



$|Cb|$ $|?Cb|$ $|\forall x?Cx|$

Determine the entailment relations between these 3 examples.

Interim summary

Standard epistemic logic: $w \vDash \varphi$ iff $w \in |\varphi|^C$

- $|\varphi|^C$ is a set of possible worlds
- $w \vDash K\varphi$ iff $\sigma(w) \subseteq |\varphi|^C$

Inquisitive epistemic logic: $s \vDash \varphi$ iff $s \in |\varphi|^I$

- $|\varphi|^I$ is an inq-proposition in inquisitive semantics
- $s \vDash K\varphi$ iff for all $w \in s, \sigma(w) \vDash \varphi$
- $K\varphi$ is always non-inquisitive (truth-conditional), but ϕ can be inquisitive
- A uniform treatment of declarative and interrogative complements, which is backward compatible with standard EL.

Wonder

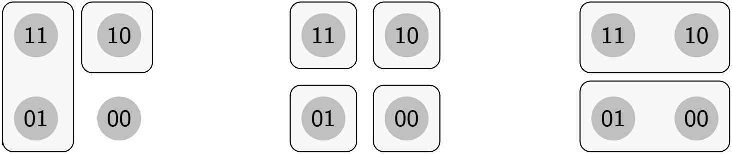
- $W\phi := \boxplus\phi \wedge \neg\Box\phi$
- Two components in the meaning of wonder:
  - agent wants to know ($\boxplus$) & agent doesn't know ($\neg\Box$)
- $\boxplus\phi$ is declarative/truth-conditional (just like $\neg\Box\phi$):
  - Support: $M, s \models \boxplus\phi \Leftrightarrow$ for all $w \in s$: for all $t \in \Sigma(w), M, t \models \phi$
  - Truth-conditions: $M, w \models \boxplus\phi \Leftrightarrow$ for all $t \in \Sigma(w), M, t \models \phi$

  $\boxplus\phi$ true in w iff all info states t in the agent's inquisitive state $\Sigma(w)$, support ϕ, equivalently $\Sigma(w) \subseteq |\phi|^I$
- An agent wonders ϕ in w iff
  - the agent inquisitive state $\Sigma(w)$ is a subset of $|\phi|^I$ ($\boxplus$)
  - the agent's information state $\sigma(w)$ not an element of $|\phi|^I$ ($\neg\Box$)
- Wonder $\mapsto$ the agent's current epistemic state does not already resolve the issue, and the agent strives to reach an information state that does.
- While $W\phi$ is truth-conditional, ϕ needs not be (in fact it cannot be)
- Account of selection properties of *wonder* (extending standard EI)

Wonder: examples

$W\varphi := \neg\Box\varphi \wedge \boxplus\varphi$

(60) Alice wonders whether Bob is coming. $\mapsto W(?Cb)$



$\Sigma(w_1)$ $\Sigma(w_2)$ $|?Cb|$

Truth conditions: $w \models W(?Cb)$ iff both conditions below hold

- $\sigma(w) \notin |?Cb|$
- $\Sigma(w) \subseteq |?Cb|$

True in w_1? No, $\sigma(w_1) \notin |?Cb|$ (ok), but $\Sigma(w_1) \not\subseteq |?Cb|$
True in w_2? Yes, both conditions satisfied

Wonder: predictions

Fact: If φ is non-inquisitive, $W\varphi$ is a contradiction

Proof: By definition $w \vDash W\varphi$ iff

- $\sigma(w) \notin |\varphi|^I$, and
- $\Sigma(w) \subseteq |\varphi|^I$

From the second condition, it follows that

- $\sigma(w) = \bigcup \Sigma(w) \subseteq \bigcup |\varphi| = \text{info}(\varphi)$

If φ is non-inquisitive, by definition $\text{info}(\varphi) \in |\varphi|^I$, and due to downward closure we should have $\sigma(w) \in |\varphi|^I$, contradicting the first condition.

(61) #Alice wonders that Bob is coming.

Exercise: wonder and know

(62) Alice wonders who the culprit is. $W(\forall x.?Cx)$

(63) Alice knows that the culprit is Bob or Charlie. $K(Cb \vee_{*} Cc)$

(64) So, Alice wonders whether the culprit is Bob or Charlie. $W(Cb \mathbin{\vee\!\!\!/} Cc)$

Prove that if (62) & (63) are true at a world, so is (64).

Prove that (62) & (63) entail (64) in the sense of inquisitive semantics.

Summary & conclusion

Standard epistemic logic: $w \vDash \varphi$ iff $w \in |\varphi|^C$

- $|\varphi|^C$ is a set of possible worlds
- $w \vDash K\varphi$ iff $\sigma(w) \subseteq |\varphi|$

Inquisitive epistemic logic: $s \vDash \varphi$ iff $s \in |\varphi|^I$

- $|\varphi|^I$ is an inq-proposition in inquisitive semantics
- $w \vDash K\varphi$ iff $\sigma(w) \in |\varphi|^I$
- $w \vDash W\varphi$ iff $\sigma(w) \notin |\varphi|^I$ & $\Sigma(w) \subseteq |\varphi|^I$
- $K\varphi$ is always non-inquisitive (truth-conditional), but ϕ can be inquisitive
- $W\varphi$ is always non-inquisitive (truth-conditional), but ϕ must be inquisitive
- A uniform treatment of declarative and interrogative complements, which is backward compatible with standard EL but extends it in a substantial way.