Modal μ-Calculus Course (Tsinghua 2024)

First homework assignment

- Deadline: 9 July, 14h20.
- Success!

Exercise 1 We write $\varphi \models \psi$ to denote that ψ is a *local consequence* of φ, that is, if for all pointed Kripke models $(\mathbb{S}, s)$ it holds that $\mathbb{S}, s \Vdash_g \varphi$ implies $\mathbb{S}, s \Vdash_g \psi$. Show that $\mu x.(x \vee \gamma) \wedge \delta \models \mu x.\gamma \wedge \delta$, for all modal (fixpoint-free)formulas γ and δ.

(In fact, the statement holds for all formulas γ and δ, but you only need to prove it in the case that γ and δ are fixpoint-free).

Exercise 2 (boolean μ-calculus) Show that the least and greatest fixpoint operators do not add expressive power to classical propositional logic, or, in other words, that the modality-free fragment of the modal μ-calculus is expressively equivalent to classical propositional logic.

Hint: you may use (with proof) the principle that $\eta x.\varphi \equiv \varphi$ if x does not occur in φ, and (without proof) the principles that $\mu x.(x \vee \gamma) \wedge \delta \equiv \mu x.\gamma \wedge \delta$, and $\nu x.(x \wedge \gamma) \vee \delta \equiv \nu x.\gamma \vee \delta$, for all formulas γ and δ.