Modal μ-Calculus Course (Tsinghua 2024)

Second homework assignment

- Deadline: 10 July, 14h20.
- Success!

Exercise 1 In this exercise you are asked to give a very short justification of your answer, but it is not needed to prove that the formulas which you provide are correct.

(a) Give a $\mu\mathtt{ML}$-formula $\xi(p,q)$ such that, for any pointed Kripke model $(\mathbb{S},s)$ we have that $\mathbb{S},s \Vdash_g \xi$ iff there is a path starting at s on which p is true infinitely often, and q is true everywhere.

(b) Give a $\mu\mathtt{ML}$-formula $\xi(p,q)$ such that, for any pointed Kripke model $(\mathbb{S},s)$ we have that $\mathbb{S},s \Vdash_g \xi$ iff there is a path starting at s on which p is true infinitely often, and q is true only finitely often.

Exercise 2 We write $\varphi \models \psi$ to denote that ψ is a *local consequence* of φ, that is, if for all pointed Kripke models $(\mathbb{S},s)$ it holds that $\mathbb{S},s \Vdash_g \varphi$ implies $\mathbb{S},s \Vdash_g \psi$. Show that $\mu x.\nu y.\,\alpha(x,y) \models \nu y.\mu x.\,\alpha(x,y)$, for all formulas α.

Exercise 3 (BONUS) Let φ,ψ be any two clean formulas of the modal μ-calculus such that ψ is free for x in φ; it will also be convenient to assume that ψ is not a subformula of φ. Show by a game semantic argument that the following so-called 'co-induction principle' holds for greatest fixpoints: if $\psi \models \varphi[\psi/x]$, then $\psi \models \nu x.\varphi$ also. (Here we write $\alpha \models \beta$ to denote that β is a *local consequence* of α, that is, if for all pointed Kripke models $(\mathbb{S},s)$ it holds that $\mathbb{S},s \Vdash_g \alpha$ implies $\mathbb{S},s \Vdash_g \beta$.)