Topological Approaches to Epistemic Logic
Homework 1

Due Wednesday, July 16, 2025, 13h00 (Beijing time)!

The homework is **individual** and **obligatory** for passing the course. BONUS question is not mandatory but it will give you extra points.

1. (30 pts) Let (X, τ) be a topological space and $A \subseteq X$. Show that A is closed in (X, τ) iff A contains all of its limit points.

   Hint: You may use the following fact:
   Fact 1: Given a topological space (X, τ) and $A \subseteq X$, A is an open set of (X, τ) iff $A = Int(A)$.

2. Given a topological space (X, τ) and $A \subseteq X$, A is called an *open domain* if $A = Int(Cl(A))$. Show that:

   (a) (20 pts) The interior of a closed set is an open domain.

   (b) (20 pts) The intersection of two open domains is an open domain.

   Hint: You may use the following facts:
   Fact 2: Given a topological space (X, τ) and $A, B \subseteq X$, if $A \subseteq B$ then $Int(A) \subseteq Int(B)$.
   Fact 3: Given a topological space (X, τ) and $A, B \subseteq X$, if $A \subseteq B$ then $Cl(A) \subseteq Cl(B)$.

3. (30 pts) Prove that $p \to \Box\Diamond p$ is valid on a topological space (X, τ) iff every closed set of (X, τ) is open.

4. (BONUS - 20 pts) Prove Proposition 7 on slide 25 of Lecture 2.