Logic and Learning

Day 1: Belief Revision

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Tsinghua Logic Summer School
June 30th–July 3rd, 2026

The problem of Belief Revision

Belief revision is a topic of much interest in theoretical computer science and logic, and it forms a central problem in research into artificial intelligence. In simple terms: how do you update a database of knowledge in the light of new information? What if the new information is in conflict with something that was previously held to be true?

Gärdenfors, Belief Revision

History: Data Bases, Theories and Beliefs

- Computer science: updating databases (Doyle 1979 and Fagin et al. 1983)
- Philosophy (epistemology):
  - scientific theory change and revisions of probability assignments;
  - belief change (Levi 1977, 1980, Harper 1977) and its rationality.

Outline

Belief revision on belief sets

Belief revision on plausibility orders

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AGM Belief Revision Model

- Names: Carlos **A**lchourrón, Peter **G**ärdenfors, and David **M**akinson.
- 1985 paper in the Journal of Symbolic Logic.
- Starting point of belief revision theory.

Belief Representation in AGM

We are talking about **beliefs** rather than **knowledge**.

In this context, the difference is that beliefs are changeable and can be false.

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Language of Beliefs in AGM

Beliefs are expressed in propositional logic:

- propositions $p, q, r, \ldots$
- connectives: negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), implication ($\rightarrow$), and biconditional ($\leftrightarrow$).

Belief Sets

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Belief set is a set of formulas that is **deductively closed**.
As such it is an abstract object.

Why are Belief Sets important?

Example

Assume Bob tells you that their beliefs include:

$$p, q.$$

Upon further inquiry it turns out that Bob also believes:

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Upon further inquiry it turns out that Bob also believes:

$$p \rightarrow \neg q.$$

Bob's beliefs are inconsistent!

Even though Bob's expressed beliefs do not include two complementary literals, $\neg q$ can be deduced from p and $p \rightarrow \neg q$.

He is committed to both q and $\neg q$, and so both are in Bob's belief set.

DEFINITION

For any set B of sentences, $Cn(B)$ is the set of **logical consequences** of B.

If φ can be derived from B by classical propositional logic, then $\varphi \in Cn(B)$.

EXAMPLE (CNTD)

Bob's expressed beliefs form the set: $B = \{p, q, p \rightarrow \neg q\}$, then $B \subset Cn(B)$, $p \wedge q \in Cn(B)$, but also $\neg q \in Cn(B)$.

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Why is it problematic to have inconsistent beliefs?

If B is an inconsistent set of formulas, then for any formula φ, $\varphi \in Cn(B)$.

In other words, we can deduce anything from inconsistent beliefs, as such they are uninteresting and uninformative!

Bob revises his beliefs

Example

Assume Bob believes: $Cn(\{p, q\})$

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Assume Bob believes: $Cn(\{p, q, r\})$

He learns, from a reliable source:

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Assume Bob believes: $Cn(\{p, q, p \to q\})$

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He learns, from a reliable source: $\neg q$

What should his new belief set be?

Bob's new belief set should be:

Option A: $Cn(\{p\})$
Option B: $Cn(\{p \to q, \neg q\})$
Option C: $Cn(\{p, q, p \to q\})$
Option D: $Cn(\{p, \neg q\})$
Option E: $Cn(\{p, p \to q\})$
Option F: $Cn(\{p, \neg q, p \to q\})$

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Assume Bob believes:

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What should his new belief set be?

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Option B: $Cn(\{p \to q, \neg q\})$

Option D: $Cn(\{p, \neg q\})$

Three Parts of Taking in New Information

What can I do to my belief set?

1. **Revision**: $B * \varphi$; φ is added and other things are removed, so that the resulting new belief set B' is consistent.
2. **Contraction**: $B \div \varphi$; φ is removed from B giving a new belief set B'.
3. **Expansion**: $B + \varphi$; φ is added to B giving a new belief set B'.

Levi Identitiy

One formal way to combine those two is to use:

Levi identity

$$B * \varphi := (B \div \neg\varphi) + \varphi.$$

Belief revision can be defined as first removing any inconsistency with the incoming information and then adding the information itself.

AGM$^{\div}$ Rationality Postulates of Contraction

1. **Closure**: $B \div \varphi = Cn(B \div \varphi)$

   the outcome is logically closed
2. **Success**: If $\varphi \notin Cn(\emptyset)$, then $\varphi \notin Cn(B \div \varphi)$

   the outcome does not contain φ
3. **Inclusion**: $B \div \varphi \subseteq B$

   the outcome is a subset of the original set
4. **Vacuity**: If $\varphi \notin Cn(B)$, then $B \div \varphi = B$

   if the incoming sentence is not in the original set then there is no effect
5. **Extensionality**: If $\varphi \leftrightarrow \psi \in Cn(\emptyset)$, then $B \div \varphi = B \div \psi$.

   the outcomes of contracting with equivalent sentences are the same
6. **Recovery**: $B \subseteq (B \div \varphi) + \varphi$.

   contraction leads to the loss of as few previous beliefs as possible
7. **Conjunctive inclusion**: If $\varphi \notin B \div (\varphi \wedge \psi)$, then $B \div (\varphi \wedge \psi) \subseteq B \div \varphi$.
8. **Conjunctive overlap**: $(B \div \varphi) \cap (B \div \psi) \subseteq B \div (\varphi \wedge \psi)$.

AGM* Rationality Postulates of Revision

1. **Closure**: $B * \varphi = Cn(B * \varphi)$
2. **Success**: $\varphi \in B * \varphi$
3. **Inclusion**: $B * \varphi \subseteq B + \varphi$
4. **Vacuity**: If $\neg\varphi \notin B$, then $B * \varphi = B + \varphi$
5. **Consistency**: $B * \varphi$ is consistent if φ is consistent.
6. **Extensionality**: If $(\varphi \leftrightarrow \psi) \in Cn(\emptyset)$, then $B * \varphi = B * \psi$.
7. **Superexpansion**: $B * (\varphi \wedge \psi) \subseteq (B * \varphi) + \psi$
8. **Subexpansion**: If $\neg\psi \notin B * \varphi$, then $(B * \varphi) + \psi \subseteq B * (\varphi \wedge \psi)$.

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Working with Sets of Formulas

Obviously, we want $B \div \varphi$ to be a subset of B that does not imply φ.

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For any set A and sentence φ the **remainder for** A **and** φ is any set C s.t.:

- C is a subset of A,
- C does does not imply φ, and
- there is no set C' such that C' does not imply φ and $C \subset C' \subseteq A$.

Contraction Remainders

Definition

For any set A and sentence φ the **remainder set** $A \perp \varphi$ is the set of inclusion-maximal subsets of A that do not imply φ.

Formally: a set C is an element of $A \perp \varphi$ just in case $C \subset A$, $C \not\models \varphi$, and there is no set C' such that $C' \not\models \varphi$ and $C \subset C' \subseteq A$.

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But:
For belief set A and any φ inconsistent with A, A revised with φ will be complete, i.e., for any formula ψ, $\psi \in A * \varphi$ or $\neg\psi \in A * \varphi$.
This means that any agent after revising their belief becomes omniscient.

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Argument:
Assume our agent has a belief set A.
Incoming information is φ, such that $\neg\varphi \in A$.

Then, for every formula ψ both $\neg\varphi \vee \psi$ and $\neg\varphi \vee \neg\psi$ are in A.
Upon contracting $\neg\varphi$ according to the above-described way,
for any formula ψ, either ψ or $\neg\psi$ are in $A \dotdiv \neg\varphi$.

Solution: Let $B \div \varphi$ be the **intersection** of some of the remainders!

Definition

A **selection function** for B is a function γ such that if $B \perp \varphi$ is non-empty, then $\gamma(B \perp \varphi)$ is a non-empty subset of $B \perp \varphi$.

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A **selection function** for B is a function γ such that if $B \perp \varphi$ is non-empty, then $\gamma(B \perp \varphi)$ is a non-empty subset of $B \perp \varphi$.

The outcome of the **partial meet contraction** is equal to the intersection of the selected elements of $B \perp \varphi$, i.e., $B \div \varphi = \cap \gamma(B \perp \varphi)$.

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Suppose that the belief set contains the sentence p: *Shakespeare wrote Hamlet*. Due to logical closure it then also contains the sentence $p \vee q$, *Shakespeare wrote Hamlet or Dickens wrote Hamlet*. The latter sentence is a mere logical consequence that should perhaps have no standing of its own.

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Discussion:

- Logical omniscience; belief sets are theories.
- 'Belief set is not what is believed, but what one is committed to believe' (Levi 1991).
- Computationally feasible: **belief base** instead of belief set.

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Those elements of the belief set that are not in the belief base are 'merely derived', i.e., they have no independent standing.

Changes are performed on the belief base. The underlying intuition is that the merely derived beliefs are not worth retaining for their own sake. If one of them loses the support that it had in basic beliefs, then it will be automatically and implicitly discarded.

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One belief set can be represented by different belief bases.
So, belief bases have more expressive power than belief sets.

Example

Alice's basic beliefs: p and q.
Bob's basic beliefs: p and $p \leftrightarrow q$.
Are Alice's and Bob's beliefs the same?
Go to: www.menti.com and enter code: 4560 3653

Example continued

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Say Alice and Bob receive and accept the information that p is false.
So, they both revise their belief bases to include the new belief $\neg p$.

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After that, Alice has the basic beliefs $\neg p$ and q, whereas Bob has the basic beliefs $\neg p$ and $p \leftrightarrow q$.

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After that, Alice has the basic beliefs $\neg p$ and q, whereas Bob has the basic beliefs $\neg p$ and $p \leftrightarrow q$.

Now, their belief sets are no longer the same:
Alice believes that q whereas Bob believes that $\neg q$.

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$$\{p, p \wedge q, p \vee q, p \leftrightarrow q\} \perp p =$$

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$$\{p, p \wedge q, p \vee q, p \leftrightarrow q\} \perp p = \{\{p \vee q\}, \{p \leftrightarrow q\}\}$$

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Belief revision on plausibility orders

Thinking in Terms of Plausibility Orders: Prior

Bob believes: $Cn(\{p, q, p \to q\})$, i.e., the state x is the most plausible.

But there are different ways in which the remaining options can be ordered.

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
			w
	y		
		z	
x			

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
	y		
		z	
			w
x			

In the above pictures, the lower the state the more plausible it is.

Bob believes: $Cn(\{p, q, p \to q\})$, i.e., the state x is the most plausible.

After revising with $\neg q$ his **posterior plausibility** changes differently depending on the **prior plausibility**.

We are looking for prior-minimal states that do not satisfy q.

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
			w
	y		
		z	
x			

Table: Option B: $Cn(\{p, \neg q\})$

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
	y		
		z	
			w
x			

Table: Option E: $Cn(\{p \to q, \neg q\})$

Thinking in Terms of Plausibility Orders

Contraction

Bob believes: $Cn(\{p, q, p \to q\})$, i.e., the state x is the most plausible.

After contracting with q, Bob has to expand his view.

We are looking for prior-minimal states that do not satisfy q.

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
			w
	y		
		z	
x			

Table: $Cn(\{p\})$

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
	y		
		z	
			w
x			

Table: $Cn(\{p \leftrightarrow q\})$

After contraction Bob's beliefs are specified by the union of his prior most plausible world and the prior most plausible word not-entailing q.

Revision and Contraction on Plausibility Orders Formally

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

Table: Plausibility order over valuations

more plausible

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p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

TABLE: Plausibility order over valuations

Definition

Let P be a set of propositions (e.g. above, $P = \{p, q\}$). A **plausibility order** is a total preorder $\leq$ over the possible truth assignments W on P. A total preorder on X is a binary relation that is:

- transitive: for all $x, y, z \in X$, if $x \leq y$ and $y \leq z$, then $x \leq z$;
- complete: for all $x, y \in X$, $x \leq y$ or $y \leq x$.

Revision and Contraction on Plausibility Orders Formally

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

Table: Plausibility order over valuations

$\downarrow$ more plausible

Let B be a belief set, φ a formula, and let $|\varphi| := \{x \in W \mid \varphi \text{ is true in } x\}$.

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p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

Table: B is determined by the most plausible world(s)

Let B be a belief set, φ a formula, and let $|\varphi| := \{x \in W \mid \varphi \text{ is true in } x\}$.

- $\varphi \in B$ iff $min_{\leq}(W) \subseteq |\varphi|$;

Revision and Contraction on Plausibility Orders Formally

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

$\downarrow$ more plausible

Table: $B * \neg p$ is determined by min world(s) with $\neg p$

Let B be a belief set, φ a formula, and let $|\varphi| := \{x \in W \mid \varphi \text{ is true in } x\}$.

- $\varphi \in B$ iff $min_{\leq}(W) \subseteq |\varphi|$;
- $\varphi \in B * \psi$ iff $min_{\leq}(|\psi|) \subseteq |\varphi|$;

Revision and Contraction on Plausibility Orders Formally

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

Table: $B \div \neg p$ is the union of the previous two

Let B be a belief set, φ a formula, and let $|\varphi| := \{x \in W \mid \varphi \text{ is true in } x\}$.

- $\varphi \in B$ iff $min_{\leq}(W) \subseteq |\varphi|$;
- $\varphi \in B * \psi$ iff $min_{\leq}(|\psi|) \subseteq |\varphi|$;
- $\varphi \in B \div \psi$ iff $min_{\leq}(|\neg\psi|) \cup min_{\leq}(W) \subseteq |\varphi|$